Problem Set 6 — Due Wednesday, February 25, 1998

Problem 1. Fully specify a TM that accepts $L = \{ww^R : w \in \{0,1\}^*\}$.

Problem 2. Problem 3.9, p. 148.

Problem 3. A *k-pebble machine* is a Turing machine with a single read-only, one-way infinite tape. The machine also has k (identical) pebbles. It can place a pebble on the tape square it is scanning, it can pick up a pebble from the square it is scanning, and it can tell whether or not the square it is scanning contains any pebbles. You can imagine that, initially, all of the pebbles are on the leftmost tape square, but as the machine computes it can push those pebbles around. Show that, for some constant k, the following is true: if L is Turing acceptable, then there is a k-pebble machines which accepts L.

Problem 4. Page 170, Problem 4.18.

Problem 5.

Part A. Show that the Turing decidable languages are closed under the star operator.

Part B. Show that the Turing acceptable languages are closed under the star operator.

Problem 6. Let $\Sigma = \{0,1\}$. Let $\Gamma \supseteq \{0,1,\#,\$,q_0,q_A\}$ be a larger alphabet. Fix a function *legal* : $\Gamma^4 \times \Gamma^4 \to \{0,1\}$. We say that a string c is *well-formed* if $c = \$c_1\$c_2\$\cdots\$c_t\$$, for some number t, where $c_j \in (\Gamma - \$)^*$ (for every $1 \le j \le t$), and $|c_j| = |c_{j+1}|$ (for every $1 \le j < t$). Let $c = \$c_1\$c_2\$\cdots\$c_t\$$ be a a well-formed string, with $|c_1| = k$. We say that c is a *computation string* if $legal(c[i..i+3], c[i+k+1..i+k+4]) = 1$[1] for each $i \in \{1,\ldots,|c|-k-4\}$. We say that c is *x-initial* if $x \in \Sigma^*$ and if $c_1 = q_0x\#^i$ for some i. We say that c is an *accepting string* if c_t contains the character q_A.

Suppose that L is Turing acceptable. Define a function *legal* (depending on the Turing machine M that accepts L) such that $x \in L$ if and only if there is an x-initial accepting computation string, c_x.

[1] By $x[l..r]$ we mean the substring of x which runs from positions l to r, where the leftmost character of x is numbered 1.