Midterm Exam

Instructions: Relax. Smile. Be happy. Then **think** about each question for a few minutes before writing down a **brief**, **correct** answer!

Bon courage!

— Phil Rogaway

Name:

E-mail:

On problem	you got	out of
1		40
2		35
3		25
Σ		100

1 Short Answer [40 points]

(1) Draw a DFA that accepts $L = \{x \in \{a,b\}^* : \ x \text{ starts and ends with different characters}\}$.

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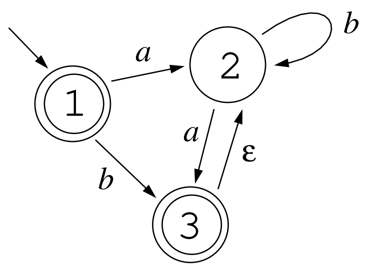
(2) Using the procedure we have seen in class, convert the following NFA into a DFA that accepts the same language.



(3) You are given a regular expression α. Describe a decision procedure (algorithm) that determines if $L(\alpha)$ contains an odd-length string.

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(4) Using the construction given in class and in your text, convert the regular expression $\alpha = (01 \cup 1)^*$ into an NFA for the same language.

(5) Complete the definition (as given in class or your book): A **context free grammar** is a 4-tuple $G = (V, \Sigma, R, S)$ where[1]

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(6) Use closure properties to show that $L = \{0^i 1^j 2^j : \ i, j \geq 0\}$ is not regular.

[1] Don't just tell me what V, Σ, R, S are called; tell me what they *are*, mathematically.

(7) Let REG be the language of all (fully parenthesized) regular expressions over the alphabet $\{0, 1\}$.

Thus sample strings in REG are:

ε
1
$((0 \circ 1) \cup 1)$

Prove that REG is CF by giving a CFG for it.

(8) With REG as defined above, prove that REG is not regular by using the pumping lemma.

2 Justified True or False [35 points]

Put an **X** through the **correct** box. Where it says "Explain" provide a **brief** (but convincing) justification. No credit will be given to correct answers that lack a proper justification. Where appropriate, **make your justification a counter-example**. Throughout, we use L and R to denote languages.

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1. If $\overline{L}$ is finite then L is regular. **True** **False**

Explain:

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2. Every regular language can be accepted by an NFA that has exactly 1,000,000 states.

Explain: **True** **False**

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3. Let R be regular and let L is a subset of R. Then L is regular.

Explain: **True** **False**

4. If L is finite then L^* is infinite. **True** **False**

Explain:

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5. Suppose L has the following property: for some number N, every string $s \in L$ having length at least N can be partitioned into $s = xyz$ such that $|y| \geq 1$ and $xy^iz \in L$ for all $i \geq 0$. Then L is regular. **True** **False**

Explain:

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6. Every CFL L can be generated by a CFG G in which every rule $A \to \alpha$ satisfies $|\alpha| \leq 2$. **True** **False**

Explain:

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7. For L regular, let $d(L)$ be the number of states in a smallest DFA for L, and let $n(L)$ be the number of states in a smallest NFA for L. Then for any regular language L, $n(L) \leq (d(L))^2$. **True** **False**

Explain:

3 A Closure Property of Regular Languages [25 points]

If L is a language over an alphabet Σ let

$$\textit{Two-Less}(L) = \{y \in \Sigma^* : \text{for some string } x \text{ having } |x| \leq 2\ , xy \in L\}.$$

Part A. *(A warm-up, just to make sure you understand the definition.)* Is one a subset of the other: L and $\textit{Two-Less}(L)$? If so, which is a subset of which?

Part B. *(A warm-up, just to make sure you understand the definition.)* Let $P = \{1^2, 1^3, 1^5, 1^7, 1^{11}, 1^{13}, \cdots\}$ be the set of prime numbers, encoded in unary. What's the shortest string in 1^* which is not in $\textit{Two-Less}(P)$?

Part C. *(Now the main problem. This has nothing to do with Parts A or B.)* Prove that if L is regular, than so is $\textit{Two-Less}(L)$. *(Describe any construction you use both in clear English and by a formal definition.)*