Problem Set 2 — Due Monday, January 25, 1999

Problem 1. Page 85, Exercise 1.12.

Problem 2. Suppose that L is DFA-acceptable. Show that the following languages are DFA acceptable, too.

(a) $\mathit{Max}(L) = \{x \in L :$ there does not exist a $y \in \Sigma^+$ for which $xy \in L\}$.

(b) $\mathit{Echo}(L) = \{a_1a_1a_2a_2 \cdots a_na_n \in \Sigma^* : a_1a_2\cdots a_n \in L\}$.

Problem 3.

(a) Show that there is an $(n+2)$-state NFA for $L_n = (\Sigma^*)0\Sigma^n$. (Take $\Sigma = \{0,1\}$.)

(b) Prove that *any* DFA for L_n requires at least 2^n states.

Problem 4. Give two substantially different proofs of the following: if L_1 and L_2 are DFA-acceptable then $L_1 \oplus L_2 = \{w : w$ is in exactly one of L_1 and $L_2\}$ is DFA-acceptable.

Problem 5. Let L be a language over Σ and define the language $\mathrm{PAL}(L) = \{x \in \Sigma^* : xx^R \in L\}$. If L is DFA-acceptable, is $\mathrm{PAL}(L)$ necessarily DFA-acceptable? Prove your answer.