Problem Set 4 — Due Monday, February 8, 1999

Problem 1. Exhibit decision procedures (algorithms) which answer the following questions. (Your algorithms need not be "efficient.")

(a) Given a DFA M, does $L(M)$ contain a string of the form $(babb \cup aa^*b)^*$?

(b) Given NFAs M_1 and M_2, is $|L(M_1)| = |L(M_2)| < \infty$?

(c) Given a regular expression α, is there a shorter regular expression which denotes the same language?

(d) Given an NFA M, is $L(M)$ a language containing only palindromes? (A palindrome is a string w such that $w^R = w$.)

Problem 2. Use closure properties and the fact that $\{a^n b^n : n \geq 0\}$ is not regular to show that the following languages are not regular.

(a) $L_a = \{a^i b^j c^k : i = j \text{ or } j = k\}$.

(b) $L_b = \{w \in \{a, b\}^* : w \text{ has more } a\text{'s than } b\text{'s}\}$.

Problem 3. Page 89, Problem 1.36.

Problem 4. Are the following propositions true or false? Give proofs or counterexamples.

(a) If $L_1 \cup L_2$ is regular and L_1 is finite, then L_2 is regular.

(b) If $L_1 \cup L_2$ is regular and L_1 is regular, then L_2 is regular.

(c) If $L_1 L_2$ is regular and L_1 is finite, then L_2 is regular.

(d) If $L_1 L_2$ is regular and L_1 is regular, then L_2 is regular.

(e) If L^* is regular then L is regular.

Problem 5. Exhibit a context free grammar for the language

$$L = \{a^n b^m : n \neq 2m\}.$$

Then describe a PDA for the same language. You do not need to be formal about the PDA; explaining how it behaves in English is fine.

Problem 6. Consider the grammar G defined by $S \to AA$, $A \to AAA \mid bA \mid Ab \mid a$.

(a) Carefully and precisely describe the $L(G)$ in an easy-to-recognize form.

(b) Is $L(G)$ regular? Prove your answer either way.

(c) Is G ambiguous? Prove your answer either way.

(d) Is $L(G)$ inherently ambiguous? Give a convincing argument either way.

Problem 7.

a. Prove that $L_a = \{a^i b^j c^k : j = \max\{i, k\}\}$ is not context free.

b. Prove that $L_b = \{b_i \# b_{i+1} : b_i \text{ is } i \text{ in binary}, i \geq 1\}$ is not context free.