# Problem Set 6 — Due Monday, March 1, 1999

**Problem 1.** Page 149, Problem 3.14.

**Problem 2.** Page 149, Problem 3.15.

**Problem 3.** For each language, indicate if it: **decidable**: the language is Turing-decidable; **acceptable**: the language is Turing-acceptable, but you can't see that the language is Turing-decidable; **not acceptable**: you can't see that the language is Turing-acceptable. If you indicate that a language is decidable, sketch the algorithm. If you indicate that a language is acceptable, sketch the TM that shows this. No further justification is necessary.

1. $L_1 = \{\langle M\rangle : M \text{ has a prime number of states}\}$
2. $L_2 = \{\langle M\rangle : M \text{ halts on some string}\}$
3. $L_3 = \{\langle M\rangle : M \text{ loops on some string}\}$
4. $L_4 = \{\langle M\rangle : M \text{ loops on every string}\}$
5. $L_5 = \{\langle M\rangle : M \text{ can enter state } q_{27} \text{ (for some input)}\}$
6. $L_6 = \{\langle M\rangle : M \text{ never enters state } q_{27} \text{ (regardless of initial input)}\}$
7. $L_7 = \{\langle M_1, M_2\rangle : L(M_1) = L(M_2)\}$
8. $L_8 = \{\langle M_1, M_2\rangle : L(M_1) \text{ is Turing-decidable}\}$
9. $L_9 = \{\langle M_1, M_2\rangle : L(M_1) \text{ is Turing-acceptable}\}$
10. $L_{10} = \{\langle M\rangle : \text{there is a regular language } R \text{ such that } R \subseteq L(M)\}$

**Problem 4.** Page 170, Problem 4.18.

**Problem 5.** Consider a *black-box-L Turing machine* $M^L$. Such a machine is just like an ordinary Turing machine, except that it has an "oracle" for the language $L$. That means that the Turing machine has a special *query tape* and, if the TM writes on that tape a string $x$ and goes into a special *query state*, $q_{\text{query}}$, then the next state will be either $q_{\text{yes}}$ or $q_{\text{no}}$, depending on whether or not $x$ is in $L$.

**Part A.** Show that $M^{A_{\text{TM}}}$ can decide any acceptable language.

**Part B.** Prove that there is a natural language $L$ that the black-box Turing machine $M^{A_{\text{TM}}}$ can *not* decide. *Hint:* Consider the "halting problem for black-box-$A_{\text{TM}}$ Turing machines."