Problem Set 8 — Due Wednesday, March 17, 1999

Problem 1. I gave a handout (available on-line) which proved Rice's theorem for the case when the emptyset does not have the property P. Finish the proof of Rice's theorem by arguing the case when the emptyset does have property P.

Problem 2. Suppose you are given a polynomial time algorithm which, on input of a Boolean formula ϕ, decides if ϕ is satisfiable. Describe an efficient procedure which finds a satisfying assignment for ϕ.

Problem 3. p. 272, problem 7.19.

Problem 4. p. 273, problem 7.21.

Problem 5. A graph $G = (V, E)$ is said to be *k-colorable* if there is a way to paint its vertices using colors in $\{1, 2, \ldots, k\}$ such that no adjacent vertices are painted the same color. When k is a number, by $kCOLOR$ we denote the language of (encodings of) k-colorable graphs. The language *3COLOR* is NP-Complete. (You can assume this.) Use this to prove that the language *4COLOR* is NP-Complete, too.

Problem 6. Let

$$D = \{\langle p \rangle : p \text{ is a polynomial (in any number of variables) and } p \text{ has an integral root.}\}$$

Prove that $3SAT \leq_{\mathrm{P}} D$.

*Note: If every NP-Complete problem reduces to Π then Π is said to be **NP-Hard.** Here you are showing that D is NP-Hard. If you remember, I claimed in lecture 1 that D is undecidable. What do you think: if Π is undecidable, is it necessarily NP-Hard? if Π is NP-Hard, is it necessarily undecidable?*