# Problem Set 1—Due Friday, October 4

**Instructions:** Write up your your solutions as clearly and succinctly as you can. Don't forget to acknowledge anyone with whom you discussed problems.

**Problem 1.** Suppose you had a choice of four algorithms to solve a problem, with the following approximate running time as a function of input size $n$:

| | |
|---|---|
| Algorithm 1 | $2^n$ seconds |
| Algorithm 2 | $20n^2$ seconds |
| Algorithm 3 | $1000n \lg n$ seconds |
| Algorithm 4 | $10000n$ seconds |

Give the range of $n$ for which each algorithm is optimal.

**Problem 2.**

**(a)** Rank the following functions by order of growth; that is, find an arrangement $g_1, \ldots, g_{30}$ such that $g_1 = \Omega(g_2), g_2 = \Omega(g_3), \ldots, g_{29} = \Omega(g_{30})$. Partition your list into equivalence classes such that $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. It is not necessary to show your work.

| | | | | |
|---|---|---|---|---|
| $\lg n$ | $\ln n$ | $\lg \lg n$ | $\lg^2 n$ | $\ln \ln n$ |
| $\ln^2 n$ | $\lg(n!)$ | $(\lg n)!$ | $n!$ | $(2n)!$ |
| $1$ | $n$ | $e^n$ | $2n^2$ | $2^n$ |
| $2^{\sqrt{n}}$ | $n \lg n$ | $4^{\lg n}$ | $n^{\lg 7}$ | $2^{n^2}$ |
| $n^{\lg \lg n}$ | $2^{2^n}$ | $n^2$ | $n^3$ | $(2/3)^n$ |
| $2^{\sqrt{\lg n}}$ | $2^{2\sqrt{\lg n}}$ | $n^{\log n}$ | $n^{0.001}$ | $e^{\sqrt{\ln n \ln \ln n}}$ |

**(b)** Give an example of a function $f(n)$ such that for all $g_i(n)$ above, $f(n)$ is niether $O(g_i(n))$ nor $\Omega(g_i(n))$.

**(c)** *(for fun)* Can you do part (b) with a monotonic function $f$? (Explain.) Drop the slowest growing pair of functions from the list. Now can you do part (b) with a monotonic function? (Explain.)

**Problem 3.** Do the "odd letters" (a, c, e, g) of problem 4–1 (p. 72), and the "odd letters" (a,c, e) of problem 4–4 (pp. 73–74).

**Problem 4.** Exercise 1.3–7 on page 16.