Problem Set 5—Due Friday, November 1

Problem 4–4 Problem 18-2 (p. 376). (Postponed problem from last week.)

Problem 5–1. Consider an algorithm Π that tests whether or not a (simple, undirected) graph is acyclic by asking questions of the form "Is there an edge between vertex i and vertex j? No other types of operations are permitted for deciding adjacency. Provide an adversary argument[1] to prove that Π must, in the worst case, query for all $n(n-1)/2$ pairs of vertices in an n-vertex graph.

Problem 5–2. Let $A_1, A_2, \ldots, A_n$ be matrices of dimensions $p_0 \times p_1, p_1 \times p_2, \ldots, p_{n-1} \times p_n$, respectively. Here is a proposed "greedy" algorithm to determine the best order in which to perform the matrix multiplications to compute $A_1 \times A_2 \times \cdots \times A_n$:

> At each step, choose the largest remaining dimension (from among $p_1, \ldots, p_{n-1}$) and multiply two adjacent matrices that share that dimension.

(a) Implementing as efficiently as you can, what is the worst case running time of this algorithm (only to determine the order in which to multiply the matrices — not to perform the actual multiplications)?

(b) Figure out whether or not the algorithm correctly minimizes the number of scalar multiplications. Prove your result either way.

Problem 5–3. Problem 16–4 (p. 326).

[1] The type of argument we used to show that $3n/2 - 1.5$ comparisons are needed to find the median, assuming comparisons only.