Problem Set 8 — Due Wednesday, December 4

Problem 8–1.

Let $G = (V, E)$ be an n-node weighted digraph, $V = \{1, \ldots, n\}$, with edge weights specified by a matrix $W = (w_{i,j})$, $w_{i,j} \in \mathbf{R}^+$. Suppose you have already computed a distance matrix $D = (d_{i,j})$, where each $d_{i,j}$ is the weight of a shortest $i \rightsquigarrow j$ path.

- **a.** Give an $O(1)$ algorithm which, on input i, j, k, ℓ, finds the length of a shortest $i \rightsquigarrow j$ path which passes through vertices k and ℓ.
- **b.** Give an $O(n)$ algorithm which, on input i and j, lists all the vertices v such that v resides on *some* shortest $i \rightsquigarrow j$ path.
- **c.** Give an $O(n \lg n)$ algorithm which, on input i and j, outputs, in order, the vertices of some shortest $i \rightsquigarrow j$ path. *Hint: start off by sorting $d_{i,x}$-values.*

Problem 8–2

a. Consider the following attempt at an algorithm for the stable marriage problem:

> *Start with an arbitrary marriage.*
> **while** *there is some Aa and Bb where A prefers b to a and b prefers A to B,* **do** *dissolve marriages Aa and Bb, and marry up Ab and Ba.*

Show (by giving a counterexample) that the above is *not* a correct algorithm — it does not necessarily terminate.

b. Consider the problem of "asexual matching" — there are n people, for n even, and each person has an ordered list of his/her preferences for living in union with each of the other $n - 1$ individuals. The problem, once again, is to find a stable matching: each individual should be matched to one and only one person, and there should exist no two couples Aa and Bb such that A prefers b and b prefers A. Show that such a matching does not necessarily exist.

Problem 8–3 Exercise 34.1–5 (p. 857).

We will cover NP-Completeness too late in the term to give you a homework problem on this. You might try the following when you are studying for your final:

Problem 8–4 (Not to hand in.) Exercise 36.5–4 (p. 960).