Problem Set 8 — Due June 2, 2000

Problem 1. Let $G = (V, E)$ be a (simple, undirected) graph with distinct edge weights. Show that G has a unique minimum-cost spanning tree.

Problem 2. Text Problem 24-1 (page 511). For this problem assume the edge weights are distinct. Define a second best MST as follows: Let T_1 be the MST of G. Then a spanning tree T_2 of G is a ***second best MST*** of G if $T_2 \neq T_1$ and for any spanning trees S other than T_1 we have that $w(S) \geq w(T_2)$.

Problem 3. Given some graph $G = (V, E)$ whose vertices are uniquely labeled 1 through $n = |V|$, we wish to determine the smallest integer k such that deleting vertices 1 through k results in a graph whose largest component has at most $n/2$ vertices. (When we delete a vertex, we also delete all edges incident to that vertex.) Give the most efficient algorithm you can to find k. State the running time of your algorithm and justify its correctness. *Hint: use union-find and build the graph backwards.*