Quiz 2

You are at the airport in Germany, about to fly back to the U.S., and in your pocket you find C Deutsch marks (DM). There is no foreign exchange facility, and so you want to spend as much of your money as possible at the duty-free shop. The duty-free shop has an infinite number of packages of each of N different types of items. You don't care about which of these you buy. Formally, the Airport Shopping Problem is the following (where "number" means a non-negative integer):

Instance: A number N, numbers $c_1, \ldots, c_N$, and a number C.

Find: Numbers $x_1, \ldots, x_n$ such that $\sum c_i x_i$ is maximized but $\sum c_i x_i \leq C$.

(Of course c_i is the cost of item i, C is the amount of money you have, and x_i is the number of packages of item i you'll buy.) The number $\sum c_i x_i$. is the *cost* of your solution. You want to find a maximal-cost solution whose cost is $\leq C$. For example, if items have costs $(c_1, c_2, c_3) = (8, 15, 6)$ and you have $C = 19$ DM, then the best solution is $(x_1, x_2, x_3) = (0, 0, 3)$: you take three of item 3 and go home with your purchases plus 1 DM, since the cost of your solution is 18.

Part A. Let $A[n, c]$ be the cost of the optimal solution if you have c DM and items have values $c_1, \ldots, c_n$. Write a recurrence relation for $A[n, c]$. Remember to specify the base case.

Part B. Write a fragment of code which calculates the $A[N, C]$-value in a reasonable amount of time. ("Reasonable" means a polynomial in N and C.) (You won't get credit on this part unless you got part A correct.)