Problem Set 5 – Due Wednesday, October 27, at 5pm

1. Let $\underline{x}$ be the 32-bit IEEE 754 floating point number closest to the real number x (assume some convention to deal with rounding). Exactly compute the 32-bit IEEE 754 product of ten and a tenth, $\underline{10}\,(\underline{0.1})$. Exactly how far off are you from the desired answer of 1? If you like, you may use a computer or online tool to assist you.

2. Show that each of the following languages are "regular" by writing them using nothing but concatenation, union, and star, applied to finite sets. Recall that the "star" operator is defined by $L^* = \bigcup_{i \in \mathbb{N}} L^i$.

   (a) The language A of binary strings whose length is divisible by two or three

   (b) The language B of all binary strings whose first two characters are the same as the last two characters. Strings in this language must have two or more characters.

   (c) The language C of all binary strings that do **not** contain two consecutive 1's.

3. Recall our definition of a group G. Using *just* those three properties we specified, carefully prove that:

   (a) The identity element is unique (any two identities are the same).

   (b) The inverse of any element is unique (any two inverses of a are the same).

4. Figure out which of the following relations $\sim$ are equivalence relations. As always, fully explain your answers.

   (a) $x \sim y$ if x and y are people who were born on the same day.

   (b) $x \sim y$ if x and y are strings that contain a common character.

   (c) $x \sim y$ if x and y are points in the plane that are equidistant to the origin.

   (d) $L \sim L'$ if L and L' are parallel lines in the plane.

5. For $a, b \in \mathbb{R}$ define $a \sim b$ if $a - b$ is an integer.

   (a) Prove that $\sim$ is an equivalence relation.

   (b) What is the equivalence class (block) containing 3? What is the equivalence class of 3.14? That is, clearly describe the sets $[3] = \{y : 3 \sim y\}$ and $[3.14] = \{y : 3.14 \sim y\}$.

   (c) If you wanted to select for each equivalence class of $\sim$ a canonical representative, or *name*, what name would you use? For example, what would be the canonical name for $[\pi]$?