Problem Set 1 — Due Friday, April 7, 2000

Recall that homework is due at 12:10 pm in the box in Eng. II, room #2131.

1. Show that $n^2 + n$ is even for any integer n.
2. Prove that $n^2 - n + 5$ is odd for any integer n.
3. Prove that $2x^2 - 4x + 3 > 0$ for any real number x
4. Prove that if n is an odd integer then there is an integer m such that $n = 4m + 1$ or $n = 4m + 3$.
5. Prove that the equations:

$$\begin{aligned} 2x + 3y - z &= 5 \\ x - 2y + 3z &= 7 \\ x + 5y - 4z &= 0 \end{aligned}$$

   have no solution.

6. Prove that there exist irrational numbers a and b such that a^b is rational.
7. Prove that there exists no smallest positive real number.
8. Write down the negation of each of the following statements in clear and concise English. Do not use expressions like "it is not the case that" in your answers. Do not worry about whether the statements are true or false.
   - Every integer is divisible by a prime.
   - There exists a, b, c such that $(ab)c \neq a(bc)$.
   - There exists an infinite set whose proper subsets are all finite.
   - There exists a planar graph which cannot be colored with at most four colors.
   - Every Eulerian graph is connected.
9. A "Venn diagram" with three overlapping circles is often used to illustrate the eight possible subsets associated with three given sets. Can the sixteen possibilities that arise with four given sets be illustrated by four overlapping circles? If so, show it; if not, indicate the maximum number of subsets that can be illustrated, and why.
10. How many different "legal configurations" are there in the Tower of Hanoi problem? Assume there are n different-sized disks.