Problem Set 2 — Due Friday, April 14, 2000

1. My computer is a 700 MHz machine and, programmed well, requires 20 clock cycles to simulate moving a ring from one peg to another in the "Towers of Hanoi" problem. How long will it take my computer to simulate finding a shortest solution to the problem when there are $n = 64$ rings? Report your answer in years, in "scientific notation".

2. Prove that $\{\rightarrow, \neg\}$ is logically complete.

3. Prove that $\{\wedge, \vee\}$ is not logically complete.

4. Translate the following sentences into a formula of sentential logic (no quantifiers): "You must file form 1040 if you are single and have made over $60,000, or if you are married and made over $80,000, or if you itemize deductions. An exception is made for those whose tax home is outside of the United States and for those who have friends high up in the White House."

5. Translate the following into a formula of the predicate calculus. "A language L that is regular will have the following property: there will be some number N (that depends on L) such that if s is a string (that is, a sequence of bits) whose length is at least N then s can be written as xyz where y is not the empty string and xy^iz is in the language L for every nonnegative integer i." (Note: you do not have to understand this statement to do this problem.)

6. Give a condition sufficient to prove that a language is **not** regular. Do this by negating the condition above. (Again, you do not have to understand this statement to do the problem.) Write your answer as a formulae of the predicate calculus.

7. Prove axiom 15, $\models \neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$, by using a truth table.

8. Prove axiom 13, $\models p \wedge (p \vee q) \Leftrightarrow p$, by using other laws on the list.

9. Three students, A, B, and C, are believed to have cheated on an examination. When they are questioned by the teacher they testify as follows:

   A: "B copied and C is innocent"

   B: "If A is guilty then so is C"

   C: "I am innocent"

   Now answer the following questions:

   (a) If A spoke the truth and B lied, who is innocent and who copied?

   (b) If everyone is innocent, who told the truth and who lied?

   (c) If C lied and B spoke the truth, who is guilty?

10. Write a Boolean circuit which realizes the function "if s then p else q." That is, your circuit has three input wires, p, q, and s, and one output wire, y.

11. Let $\mathsf{mux}(p_{00}, p_{01}, p_{10}, p_{11}, x_1, x_0) = p_{x_1 x_0}$.

    (a) Draw a circuit that computes mux using the above only and, or, and not gates.

    (b) Revise your circuit so that it uses only "nand" gates.