Problem Set 3 — Due Friday, April 21, 2000

1. In a survey of 260 college students, the following data were obtained:

   64 had taken a mathematics course,
   94 had taken a computer science course,
   58 had taken a business course,
   28 had taken both a mathematics and a business course,
   26 had taken both a mathematics and a computer science course,
   22 had taken a computer science and a business course, and
   14 had taken all three types of courses.

   (a) How many students were surveyed who had taken none of of the three types of courses?
   (b) Of the students surveyed, how many had taken only a computer science course?

2. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.

   (a) $A \in B \wedge B \in C \implies A \in C$
   (b) $A \subseteq B \wedge B \subseteq C \implies A \subseteq C$
   (c) $A \subsetneq B \wedge B \subsetneq C \implies A \subsetneq C$
   (d) $A \in B \wedge B \subseteq C \implies A \in C$

3. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.

   (a) $C \in \mathcal{P}(A) \iff C \subseteq A$
   (b) $A \subseteq B \iff \mathcal{P}(A) \subseteq \mathcal{P}(B)$
   (c) $A = \emptyset \iff \mathcal{P}(A) = \emptyset$

4. Which of the following conditions imply that $B = C$? In each case, either prove or give a counterexample.

   (a) $A \cup B = A \cup C$
   (b) $A \cap B = A \cap C$
   (c) $A \oplus B = A \oplus C$
   (d) $A \times B = A \times C$

5. Suppose that A, B and C are sets. For each of the following statements either prove it is true or give a counterexample to show that it is false.

   (a) $A \setminus (B \cup C) = (A \setminus B) \cup (A \setminus C)$
   (b) $(A \setminus B) \times C = (A \times C) \setminus (B \times C)$
   (c) $(A \oplus B) \times C = (A \times C) \oplus (B \times C)$
   (d) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

6. Write a shortest regular expression for the complement (relative to $\{0,1\}^*$) of 00^*11^*.

7. Write a regular expression for the language which is the set of all nonempty strings over $\{a,b\}$ that start and end with the same character.