Problem Set 4 — Due Friday, April 28, 2000

1. Draw a DFA that recognizes the language $L = \{a_1 \ldots a_n :$ each a_i is a bit and $a_i = 1$ for all odd $i\}$.
2. Make a regular expression for the language of the previous problem.
3. Draw a DFA that recognizes the language

$$L = \{x \in \{0,1\}^* : \text{the substring “000” occurs an even number of times in } x\}$$

4. Determine which of the following relations R is reflexive, symmetric, and transitive.
   (a) xRy if x and y are people who were born on the same day.
   (b) xRy if x and y are strings which contain a common character.
   (c) xRy if x and y are people and there exists a country C such that x has been to country C and y has been to country C.
   (d) xRy if x and y are real numbers which are equidistant to the origin.
5. For $a, b \in \mathsf{R}$ define $a \sim b$ if $a - b \in \mathsf{Z}$.
   (a) Prove that $\sim$ defines an equivalence relation on Z.
   (b) What is the equivalence class of 5? What is he equivalence class of $5\frac{1}{2}$?
   (c) What is the quotient set determined by this equivalence relation?
6. Define $g : \mathsf{Z} \to \mathsf{Z}$ by $g(x) = 2x^2 + 7x$. Is g onto? Is g one-to-one? Prove your answers.
7. Give an example of function $\mathsf{N} \to \mathsf{N}$ which is
   (a) one-to-one but not onto;
   (b) onto but not one-to-one;
   (c) neither one-to-one nor onto;
   (d) both one-to-one and onto.

   Now repeat the exercise, to the extent possible, for a function with domain and range of $\{1, 2, \ldots, 10\}$.
8. For each of the following, find the largest subset A of R such that the given formula for $f(x)$ is a function with domain A. Give the range of f in each case.
   (a) $f(x) = 1/(x-3)$
   (b) $1/\sqrt{x-3}$
9. Let $f : \mathsf{R} \to R$ be defined by $f(x) = x + \lfloor x \rfloor$. Find the range of f.
10. Let $f(x) = x \lg x$. Compute $f^{-1}(10)$ to 6 decimal places of accuracy.