Problem Set 5 — Due Friday, May 5, 2000

1. Sort the following functions into groups $G_1, G_2, \ldots$ such that $f, g \in G_i$ if $f \in \Theta(g)$, and $f \in G_i \implies f \in O(g)$ for each $g \in G_i$. (That is, the slowest growing functions are in the first group; then the next slowest growing functions; and so forth.)

$5n \lg n$	$6n^2 - 3n + 7$	1.5^n	$\lg n^4$	13463
$-15n$	$\lg\lg n$	$9n^{0.7}$	$n!$	$n + \lg n$
$\sqrt{n} + 12n$	$\lg n!$	$\log n$	e^n	2^n

2. Compute the $\Theta(\cdot)$-running time for the following code fragment. Assume that `S` takes unit time to run.

```
for i = 1 to n do
  for j = 1 to i do
    for  k = 1 to 100 do
       for m = j to j+10 do
          S
```

3. Is the following statement true or false? (Give a proof or counterexample.)

   for every pair of functions f and g, either $f \in O(g)$ or $g \in O(f)$.

4. Prove that if $f_1 \in \Theta(g)$ and $f_2 \in \Theta(g)$ then $f_1 + f_2 \in \Theta(g)$.

5. Five students live within the 1 mile $\times$ 1 mile subdivision of Macy's Ranch. Show that some pair of them live within 0.71 miles of each other.

6. In any list of 100 natural numbers, prove that there must always exist a run of consecutive numbers whose whose sum is divisible by 100.

7. In a gathering of 30 people, there are 104 different pairs of people who know each other.

   (a) Show that some person must have at least 7 acquaintances.

   (b) Show that some person must have fewer than 7 acquaintances.

8. Determine, with justification, whether each of the following sets is finite, countably infinite, or uncountable:

   (a) $\mathsf{R} - \mathsf{Q}$

   (b) $3\mathsf{Z} - 2\mathsf{Z}$ (where $i\mathsf{Z}$ denotes the set of all integral multiples of i)

   (c) The set of all strings over $\{0, 1\}$

   (d) The set of all languages over $\{0, 1\}$