Problem Set 8 — Due Friday, June 2, 2000

As you do these counting exercises, when you get a number that isn't too huge, please give it to me as an explicit number. This is fun for you, makes things easier to grade, and lets you exercise that over-priced calculator mom and dad bought you.

1. How many one-to-one functions are there from 4-byte words to 4-byte words?

2. There are 90 students in a class learning about counting. One after another, five students quietly slip out the back door. In how many ways can this exodus occur?

3. (a) In how many ways can ten boys and four girls sit in a row?
   (b) In how many ways can they sit in a row if the boys are to sit together and the girls are to sit together?
   (c) In how many ways can they sit in a row if the girls are to sit together?
   (d) In how many ways can they sit in a row if *just* the girls are to sit together?

4. Four cats and five mice enter a race. The mice are clearly superior; they place first, second, and third. In how many ways can this happen?

5. How many permutations over the letters `a`, `b`, `c`, `d`, `e`, `f`, `g` contain neither the pattern `bge` nor the pattern `eaf`?

6. (Leading zeros are not permitted in either part of this question)
   (a) How many seven-digit number have no repeated digits?
   (b) How many seven-digit number with no repeated digits contain a 3 but not a 6?

7. A group of people is comprised of six from Nebraska, seven from Idaho, and eight from Louisana.
   (a) In how many ways can a committee of six be formed with two people from each state?
   (b) In how many ways can a committee of seven be formed with at least two people in each state?

8. Eve has ten apples, and plans to give at most three of them to Adam. How many ways can she do this?

9. How many five-card hands dealt from a standard deck of 52 playing cards are all of the same suit? I you deal out a random hand, what is the probability that it will have this property?

10. A woman has 9 close friends.
    (a) In how many ways can she invite six of them to dinner?
    (b) Repeat (a) if two of her friends are divorced, hate each other, and can not both be invited.
    (c) Repeat (a) if the friends consist of three single people and three married couples and if a husband or wife is invited, the spouse must be invited, too.