Problem Set 9 — Due Friday, June 9, 2000

1. Suppose that Phil asks 5 multiple choice questions, where there are five possibilities (ie., a, b, c, d, e) for each of the questions. Phil grades the questions: 4 points for a correct answer, 0 points for an absent answer, and -1 points for an incorrect answer. Suppose that Joe (who's pretty clueless) randomly guesses on all 5 questions. What is his expected score?

   Now suppose that Phil (kind person that he is) decides to give everyone a score of *at least* zero on the multiple choice problem. (That is, you get a zero even if you earn a total that is a negative number). Now what will be Joe's expected score?

2. Consider the following one-person game. Alice initially has \$300. A dealer has a deck of 52 cards, perfectly shuffled. The dealer turns over the cards one-by-one. If at some point Alice has \$$x$ then, just before the next card is turned over, Alice can bet any amount of money in $[0, x]$. The bet is "red" or "black". If the card turned over has the color Alice said, then Alice wins \$$x$; otherwise, she loses \$$x$.

   (a) Describe a strategy such that Alice is *guaranteed* to end the game with \$600.

   (b) Describe a strategy such that Alice is *guaranteed* to end the game with than \$800.

   (c) *(Thought problem—not graded—for wiz-kids only.)* What's the most money that Alice can be certain to win?

3. What's the chance that Alice, Bob, Charlie, and Dana were are all born on different days of the week? Assume that each was born on a random day of the week.

4. A monkey, Monika, throws darts at a dart-board having five concentric circles. The circles have radiuses of 10, 20, 30, 40, and 50 cm. Half the time Monika misses the dart-board completely—watch out—while the other half of the time she manages to get the dart to land on the the dart-board, but she has no particular control about where on the dart-board it lands.

   Describe a probability space (that is, a sample space Ω and a probability measure μ) that models what is going on.

   Under your model, what is the probability that Monika gets 3 or more bull's-eyes in 5 throws? (A "bull's-eye" means that the dart lands in the centermost circle.)

5. Two graphs $G = (V, E)$ and $G' = (V', E')$ are *isomorphic* if they are the same, up to the names of the vertices: that is, if there exists a bijection $\pi : V \to V'$ such that $\{x, y\} \in E$ iff $\{\pi(x), \pi(y)\} \in E'$. Let us call two graphs "fundamentally different" if they are not isomorphic. How many fundamentally different graphs are there on four vertices?

6. Let $G = (V, E)$ be a graph. A *clique of size* k in G is a set of vertices $K \subseteq V$ such that $|K| = k$ and all pairs of vertices in K are adjacent. Let G be a graph on 100 nodes. Describe an algorithm which finds the largest clique in G. Estimate how long your algorithm would take to run.

7. Drop four vertices onto the plane. Now for each of the $\binom{4}{2}$ potential edges, include that edge with probability $1/2$. What is the probability that the graph you've just produced has a clique of size 3 (such a clique is called a *triangle*)?