Midterm Exam

Instructions: Please ask me if you don't understand anything. Please show your work. Don't worry. Be happy!

Name — in Thai

Name – in English

Nickname — in Thai

ID Number

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1. *(5 points)* Complete the definition, being mathematically exact:

   $f(n)$ is $\Theta(n \lg n)$ if there exists

2. *(5 points)* Algorithm A takes an input of size n. With probability 1/2: it does $5n$ operations and then calls itself on an input of size $9n/10$. With probablity 1/2: it does $5n$ operations and then calls itself on an input of size $n-1$. Write a recurrence relation for the expected running time, $T(n)$, of this algorithm. Do not solve.

3. *(20 points)* Solve these recurrence relations. Give your answers in $\Theta(\cdot)$ notation. Assume that $T(n)$ is $\Theta(1)$ for sufficiently small n.

   A. $T(n) = 3T(n/2) + n$

   B. $T(n) = 2T(n/2) + 5n$

   C. $T(n) = T(n-1) + 1/n$

   D. $T(n) = T(n-1) + 2n$

E. $T(n) = T(n-1) + \lg n$

4. *(20 points)* In lecture we gave an analysis of the determinisitic selection algorithm in which we treated the input as an array of columns of 5 elements, and we saw the resulting analysis gives a bound of $16n$ comparisons. Think about the corresponding algorithm which uses columns of 9 elements, not 5. **Write a recurrence relation** for this algorithm, and then **solve it.** Use the fact that we can find the median of 9 elements in 14 comparisons.

5. *(20 points)* Answer these questions **True** or **False**. Please give a few words as a reason.

   A. Given a set of n points in the plane, we have seen how to find a closest pair of them in $\Theta(n)$ time.

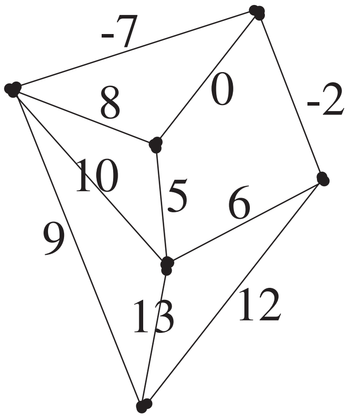
   B. QUICKSORT runs in worst-case $\Theta(n \lg n)$ time.

   C. Given a list of n real numbers, we can find the $\lceil n/3 \rceil$-rd biggest one in worst-case $\Theta(n)$ time.

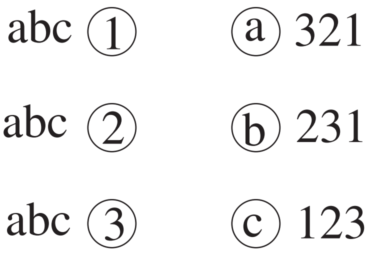
   D. If x is a string of length n and y is a string of length m we have seen how to find the longest common subsquence of x and y in $\Theta(n+m)$ time.

E. Any comparison-based algorithm to sort 5 numbers must make at least 7 comparisons.

6. (5 points) Run Kruskal's algorithm to find a Minimum Spanning Tree (MST) for the following weighted graph. **Darken** the edges of your MST.



7. *(5 points)* Run the "proposal algorithm" ("man proposes, woman disposes") on the following input to find a stable marriage. Show who, in the end, is married to whom. When there is more than one unmarried man, have the first man listed (the lowest-numbered man) be the one who proposes.



MEN

8. *(5 points)* The following 10 actvities would like to use CSB 100 tomorrow, at the indicated times. Run the scheduling algorithm we described in class to find a schedule which maximizes the number of activites which get use of the room. Circle the activites which the algorithm selects.

7:00–10:00	10:30–11:00
8:00–9:00	10:00–11:30
8:30–9:30	12:00–1:00
9:00–10:30	12:30–2:00
10:00–12:00	1:00–3:00

9. (25 points) Recall the shuffle problem from Homework 3:

> Given three strings, $x = x_1 \ldots x_N$, $y = y_1 \ldots y_M$, and $z = z_1 \ldots z_{N+M}$, figure out if z is a shuffle of x and y (i.e., can you color the letters of z red or blue such that the red subsequence of z is x and the blue subsequence of z is y)?

We solved this problem using an $(N+1) \times (M+1)$ Boolean matrix, A. For $0 \leq n \leq N$ and $0 \leq m \leq M$ we said $A[n, m]$ is **true** iff $z_1 \ldots z_{n+m}$ is a shuffle of $x_1 \ldots x_n$ and $y_1 \ldots y_m$.

A. Fill in the values of table A when $x = ab$, $y = abb$, and $z = abbab$.

B. For $n, m \geq 1$, write a formula which says how to calculate $A[n, m]$ in terms of $x_1 \cdots x_N$, $y = y_1 \cdots y_M$, $z = z_1 \cdots z_{N+M}$, and other $A[n', m']$-values, where $n' \leq n$, $m' \leq m$, $(n', m') \neq (n, m)$.

$A[n, m] =$

C. Given $x = x_1 \cdots x_N$, $y = y_1 \cdots y_M$, and $z = Z_1 \cdots z_{N+M}$, write code that fills in the correct values for $A[0, m]$, for $0 \leq m \leq M$ (that is, row 0). Your code should run in $\Theta(M)$ time.

D. Suppose we want to save space and use a Boolean table which is smaller than $(N+1) \times (M+1)$. Describe how to solve the problem in $O(M)$ space (and $O(MN)$ time).

E. Describe how to solve the problem in $M + O(1)$ space (and $O(MN)$ time)?