Problem Set 4—Due Friday, 30 July 1999

Problem 11 You are given two arrays, A and B, each containing n integers, and a target number T. Your goal is to find indices i and j such that $A[i]+B[j]=T$ if such indices exist. Describe how you can use hashing to efficiently solve this problem. Discuss the running time of your algorithm under the uniform hashing assumption, assuming that you can hash in constant time.

Problem 12 Describe a greedy algorithm for making change of n baht using coins of denominations 1 baht, 5 baht, and 10 baht. Then prove that this algorithm is correct—it always uses a minimum number of coins possible.

Problem 13 In the WOMEN'S DORMITORY PROBLEM there are n women, where n is even, and we want to assign them to rooms having two women per room. Each woman i has a "preference lists", L_i, which is a string of length $n-1$ having all the numbers $\{1,\ldots,n\}-\{i\}$, in some order. We say that woman i prefers j to k if j appears to the left of k in the string L_i. A stable matching for the WOMEN'S DORMITORY PROBLEM is an assignment of the women into rooms, two women per room, such that if x and x' share a room and y and y share a room then either x prefers x' to y or y prefers y' to x—that is, no pair of women x and y are inclined to go off and get a new room for the two of them. Prove the following: sometimes there doesn't exist a stable matching for this problem. An example will suffice to show this.