EAD210A- Numerical Methods _ Rao Vemuri

Homework Assignment 1. (Due 7 October 2004)

Programming Problem.

1. (10 pts) Do Problem 1 in Section 1.9, page 104 of the text. Use Matlab, if necessary, to plot the curves. (10 pts)

Paper-Pencil Problems

2. Verify the validity of Eq. (1.2-2a) on page 29
3. Prove the inequality (1.2-7) on page 35
4. Do Problem 3 on page 104.
5. Let $\prod_N([a,b])$ is the set of all polynomials of degree N or less than N. Show that $\prod_N([a,b])$ is a vector space of dimension N
6. (5 pts) Given, for N=2

x_i	0	1	3
f_i	1	3	2

Find P(2) where P ε $\prod_2$ and $P(x_i) = f_i$, for i = 0, 1, 2.

7. Let $L_i(x)$ be the Lagrange polynomials defined on page 29.
Let $c_i := L_i(0)$
Show that

$$\sum_{i=0}^{N} c_i x_i^j = \begin{cases} 1, for - j = 0 \\ 0, for - j = 1,2,..., N \\ (-1)^N x_0 ... x_N, for - j = (N+1) \end{cases}$$

8. Estimate the error in the approximation of f(x) = sin x with $x_i = \frac{\pi i}{10}, i = 0,1,2,3,4,5.$
N=5.
9. (a) Interpolate the function ln(x) by a quadratic polynomial at x = 10, 11, 12.
(b) Estimate the error committed at x = 11.1

10. (a) (10+5 pts)The Bessel function of order zero

$$J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x \sin t) dt$$

is to be tabulated at equidistant arguments $x_i = x_0 + ih$, I = 0,1,2,… How small must the increment *h* be chosen so that the interpolation error remains below 10^{-6}, if linear interpolation is used?

(b) What is the behavior of the maximal interpolation error

$$\max_{0 \le x \le 1} | P_N(x) - J_0(x) |$$

as n → ∞, if $P_N(x)$ interpolates $J_0(x)$ at $x = x_i^N := i / N, i = 0,1,..., N$? Hint: It suffices to show that $| J_0^k(x) | \le 1$ for k = 0, 1,…