## A Bayesian Network



$$
\begin{aligned}
& \operatorname{Pr}(\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathrm{d}, \mathbf{e})=\operatorname{Pr}(\mathbf{a}) \operatorname{Pr}(\mathrm{b} \mid \mathrm{a}) \operatorname{Pr}(\mathbf{c} \mid \mathbf{a}, \mathrm{b}) \operatorname{Pr}(\mathrm{d} \mid \mathbf{a}, \mathrm{b}, \mathrm{c}) \operatorname{Pr}(\mathbf{e} \mid \mathbf{a}, \mathrm{b}, \mathrm{c}, \mathrm{~d}) \\
& =\operatorname{Pr}(a) \operatorname{Pr}(b \mid a) \operatorname{Pr}(c \mid a) \operatorname{Pr}(d \mid b, c) \operatorname{Pr}(e \mid c)
\end{aligned}
$$

## Links Encode Conditional Probabilities



M: $\operatorname{Pr}\left(b=b_{j} \mid a=a_{i}\right)$

$M: \operatorname{Pr}\left(b=b_{j} \mid a=a_{i}, c=c_{k}\right)$

## Conditional Independence

Equivalent statements:

- b adds no useful information about a, given $S$
- $\operatorname{Pr}(\mathbf{a} \mid \mathrm{b}, \mathrm{s})=\operatorname{Pr}(\mathrm{a} \mid \mathrm{s})$
- Cl ( $\mathrm{a}, \mathrm{S}, \mathrm{b}$ )
- $\mathrm{Cl}(\mathrm{b}, \mathrm{S}, \mathrm{a})$
- a and b "d-separated" by S in graph


## Separation in Graphs: 1, 2

$x$ blocks the path from $a$ to $b$ :


## Separation in Graphs: 3

$x$ unblocks the path from $a$ to $b$ :


## D-Separation

Defn: Variables $\boldsymbol{a}$ and $\boldsymbol{b}$ are $d$-separated by S iff every undirected path from $a$ to $b$ is blocked by a variable in S and no path is unblocked.

$a$ and $b$ are d-separated by $\{w, x$ \} but by no other subset of $\{w, x, y, z\}$

## Direction Matters

IF the sprinkler was on last night
THEN there is suggestive evidence (0.9) that the grass will be wet this morning.

IF the grass is wet this morning
THEN there is suggestive evidence (0.8) that it rained last night.


## Reasoning Patterns

Asymmetry of probabilistic dependence requires distinction between predictive and diagnostic inference.
predictive = causal diagnostic = evidential
Intercausal relations specify interactions among causes of a common effect.


## "Explaining Away"



## Intercausal Reasoning



## Schematic Action Model



- $A$ is $C l$ of both $Q$ and $R$ given $S \cup B$.
- Express effect as $\operatorname{Pr}(\mathbf{S} \mid A, B)$, often simplifiable to [S |A] using:
- canonical models
- ceteris paribus clause

