Adversary Search

- Ref: Chapter 5

Games & A.I.

- Easy to measure success
- Easy to represent states
- Small number of operators
- Comparison against humans is possible.
- Many games can be modeled very easily, although game playing turns out to be very hard.

2 Player Games

- Requires reasoning under uncertainty.
- Two general approaches:
  - Assume nothing more than the rules of the game are important - reduces to a search problem.
  - Try to encode strategies using some type of pattern-directed system (perhaps one that can learn).

Search and Games

- Each node in the search tree corresponds to a possible state of the game.
- Making a move corresponds to moving from the current state (node) to a child state (node).
- Figuring out which child is *best* is the hard part.
- The *branching factor* is the number of possible moves (children).

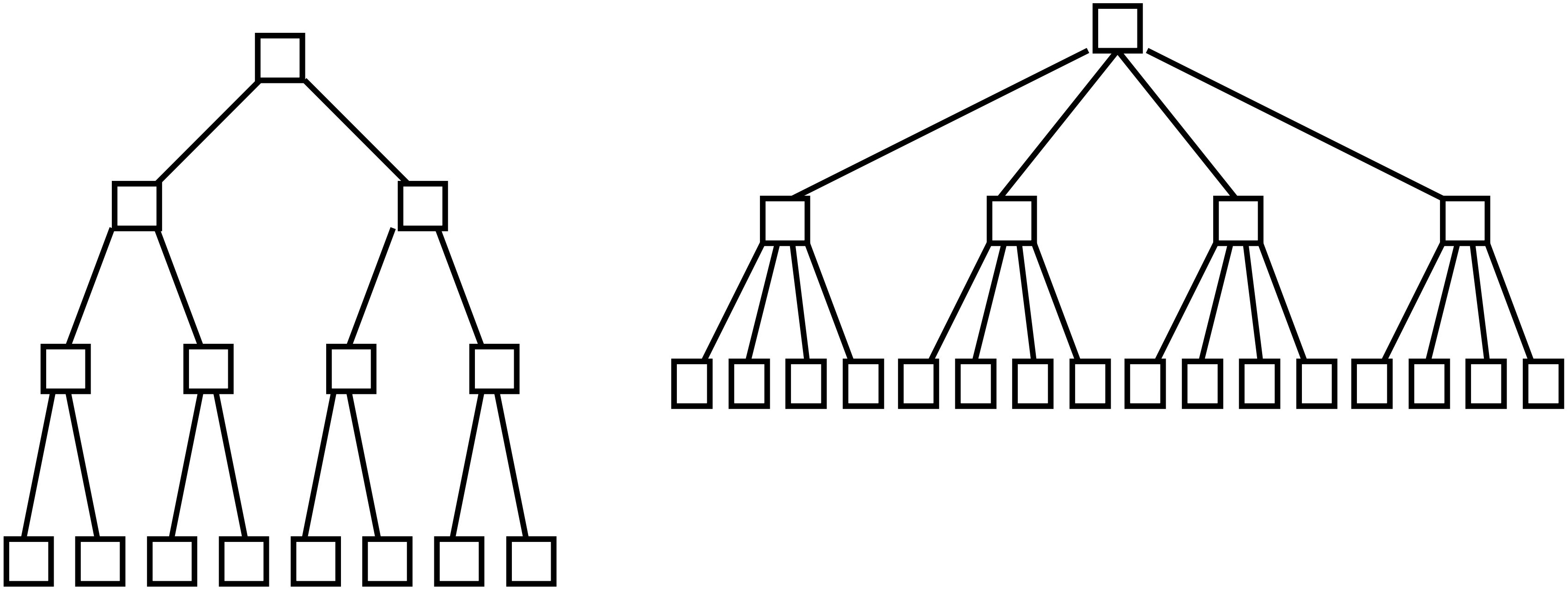
Search Tree Size

- For most interesting games it is impossible to look at the entire search tree.
- Chess:
  - branching factor is about 35
  - typical match includes about 100 moves.
  - Search tree for a complete game: 35^{100}

Heuristic Search

- Must evaluate each choice with less than complete information.
- For games we often evaluate the game tree rooted at each choice.
- There is a tradeoff between the number of choices analyzed and the accuracy of each analysis.

Game Trees



Plausible Move Generator

- Sometimes it is possible to develop a move generator that will (with high probability) generate only those moves worth consideration.

- This reduces the branching factor, which means we can spend more time analyzing each of the plausible moves.

Recursive State Evaluation

- We want to rank the plausible moves (assign a value to each resulting state).
- For each plausible move, we want to know what kind of game states could follow the move (Wins? Loses?).
- We can evaluate each plausible move by taking the value of the *best* of the moves that could follow it.

Assume the adversary is good.

- To evaluate an adversary’s move, we should assume they pick a move that is good for them.
- To evaluate how good their moves are, we should assume we will do the best we can after their move (and so on…)

Static Evaluation Function

- At some point we must stop evaluating states recursively.
- At each leaf node we apply a *static evaluation function* to come up with an estimate of how good the node is from our perspective.
- We assume this function is not good enough to directly evaluate each choice, so we instead use it deeper in the tree.

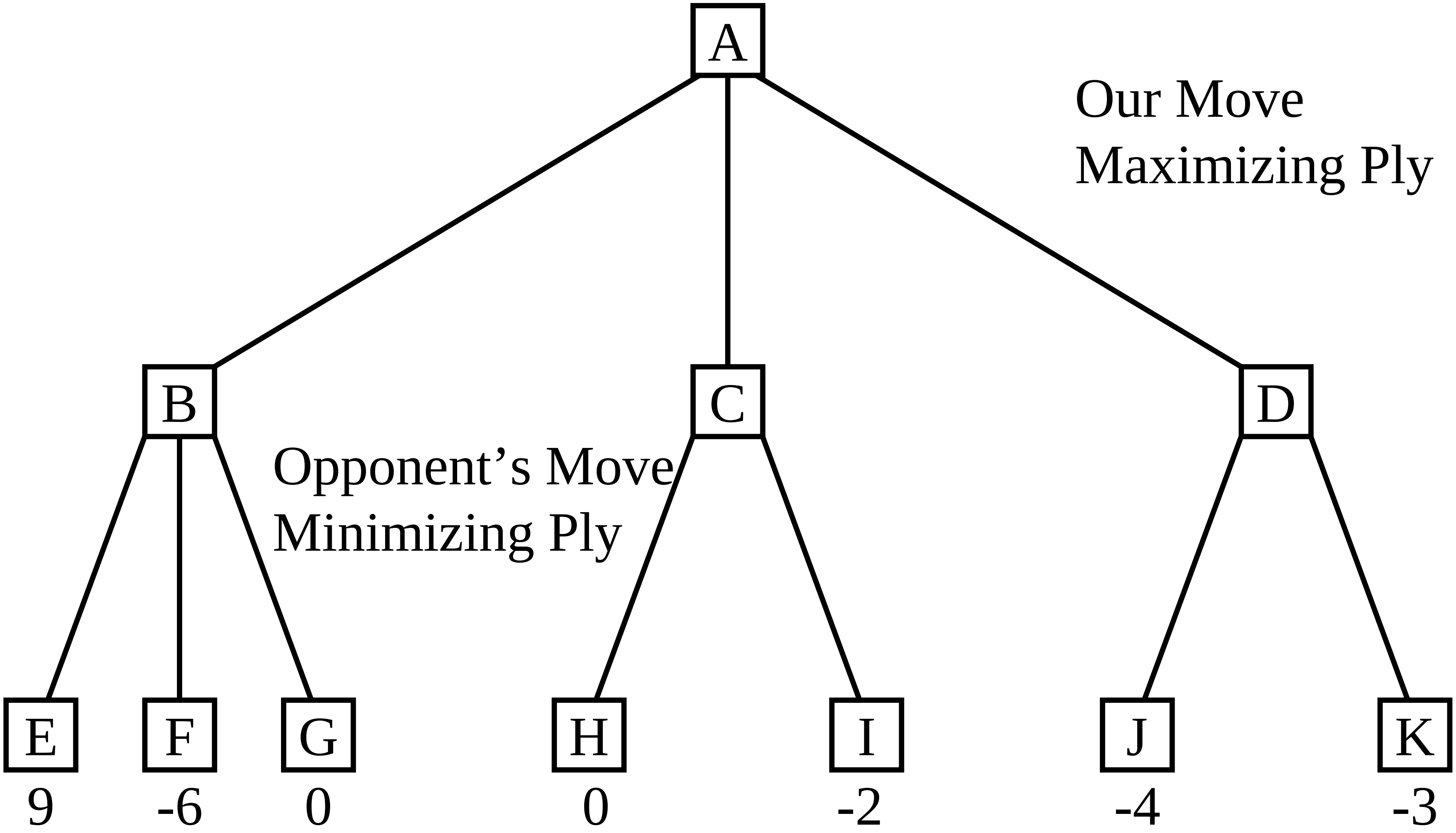
Example evaluation functions

- Tic-Tac-Toe: number of rows, columns or diagonals with 2 of our pieces.
- Checkers: number of pieces we have - the number of pieces the opponent has.
- Chess: weighted sum of pieces:
  - king=1000, queen=10, bishop=5, knight=5, ...

Minimax

- Depth-first search with limited depth.
- Use a static evaluation function for all leaf states.
- Assume the opponent will make the best move possible.

Minimax Search Tree



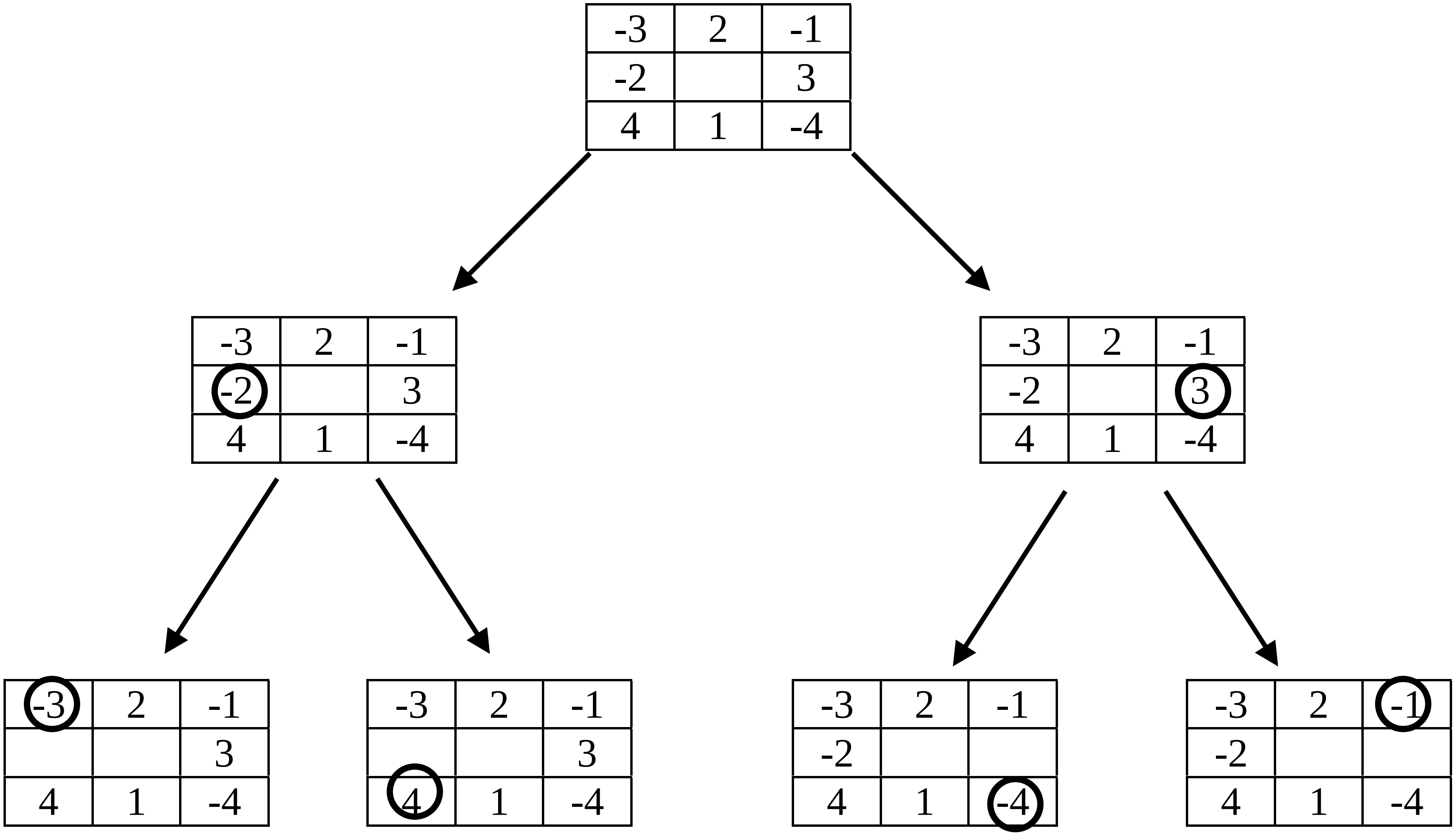
Minimax Algorithm

```
Minimax(curstate, depth, player):
  If (depth==max)
      Return static(curstate,player)
  generate successor states s[1..n]
  If (player==ME)
      Return max of Minimax(s[i],depth+1,OPPONENT)
  Else
      Return min of Minimax(s[i],depth+1,ME)
```

The Game of MinMax

-3	2	-1
-2		3
4	1	-4

- Start in the center square.
- Player MAX picks any number in the current row.
- Player MIN picks any number in the resulting column.
- The game ends when a player cannot move.
- MAX wins if the sum of numbers picked is > 0.



-3	2	-1
-2		3
4	1	-4

-3	2	-1
-2		3
4	1	-4

-3	2	-1
-2		3
4	1	-4

-3	2	-1
		3
4	1	-4

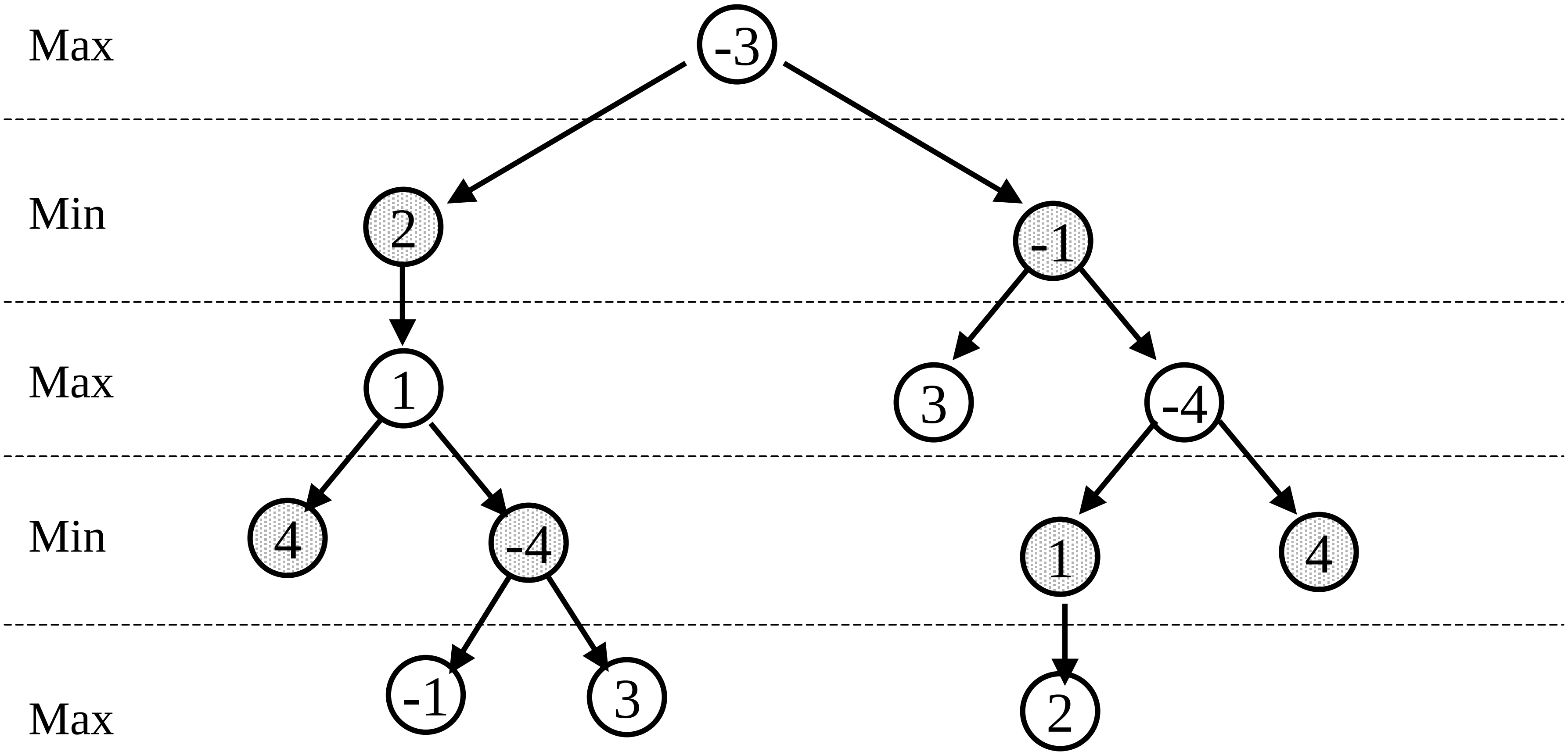
-3	2	-1
		3
4	1	-4

-3	2	-1
-2		
4	1	-4

-3	2	-1
-2		
4	1	-4

-3	2	-1
		3
4	1	-4

MAX’s Turn

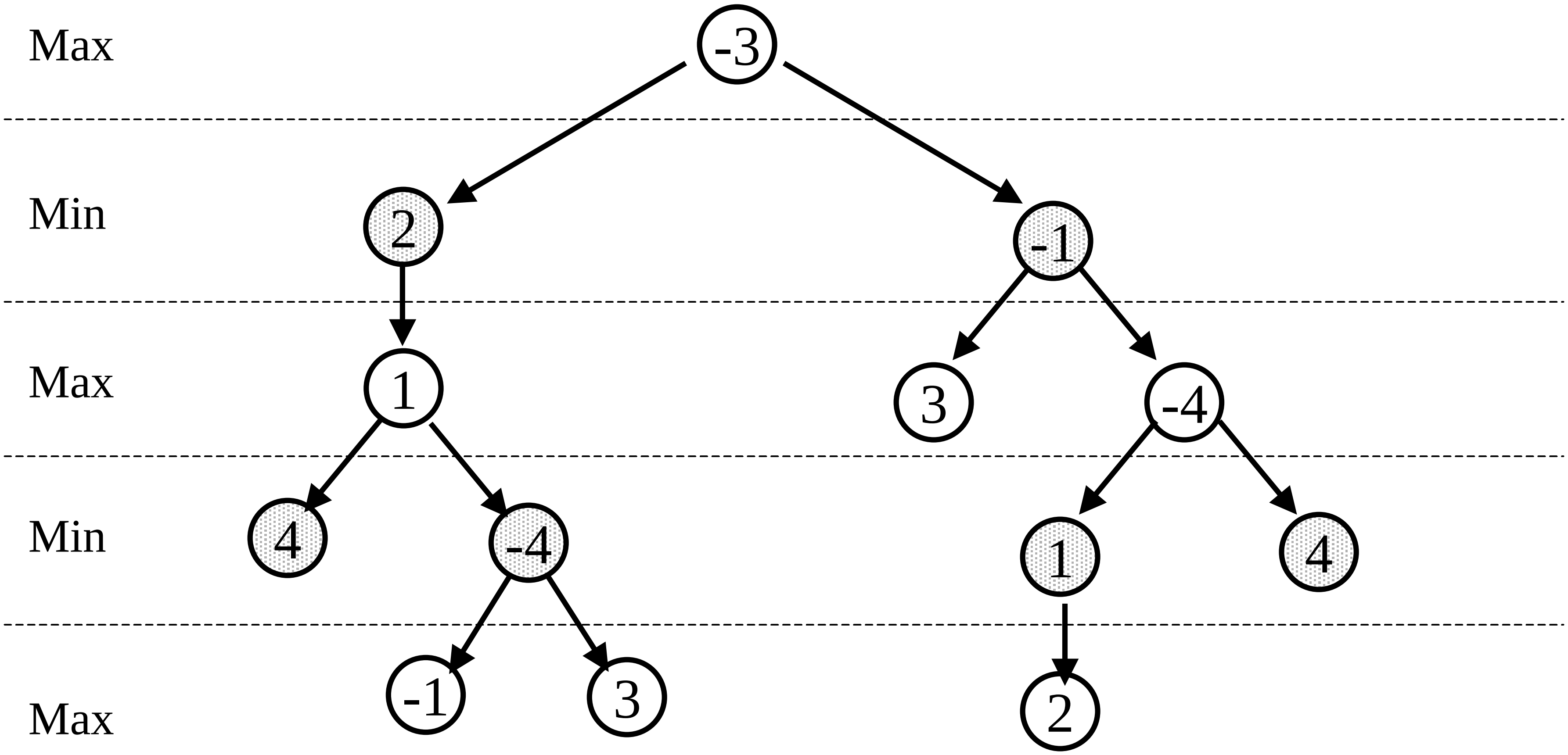


Pruning

- We can use a branch-and-bound technique to reduce the number of states that must be examined to determine the value of a tree.

- We keep track of a lower bound on the value of a maximizing node, and don’t bother evaluating any trees that cannot improve this bound.

Pruning in MinMax



Pruning Minimizing Nodes

- Keep track of an upper bound on the value of a minimizing node.
- Don’t bother with any subtrees that cannot improve (lower) the bound.

Minimax with Alpha-Beta Cutoffs

- Alpha is the lower bound on maximizing nodes.
- Beta is the upper bound on minimizing nodes.

- Both alpha and beta get passed down the tree during the Minimax search.

Usage of Alpha & Beta

- At minimizing nodes, we stop evaluating children if we get a child whose value is less than the current lower bound (alpha).

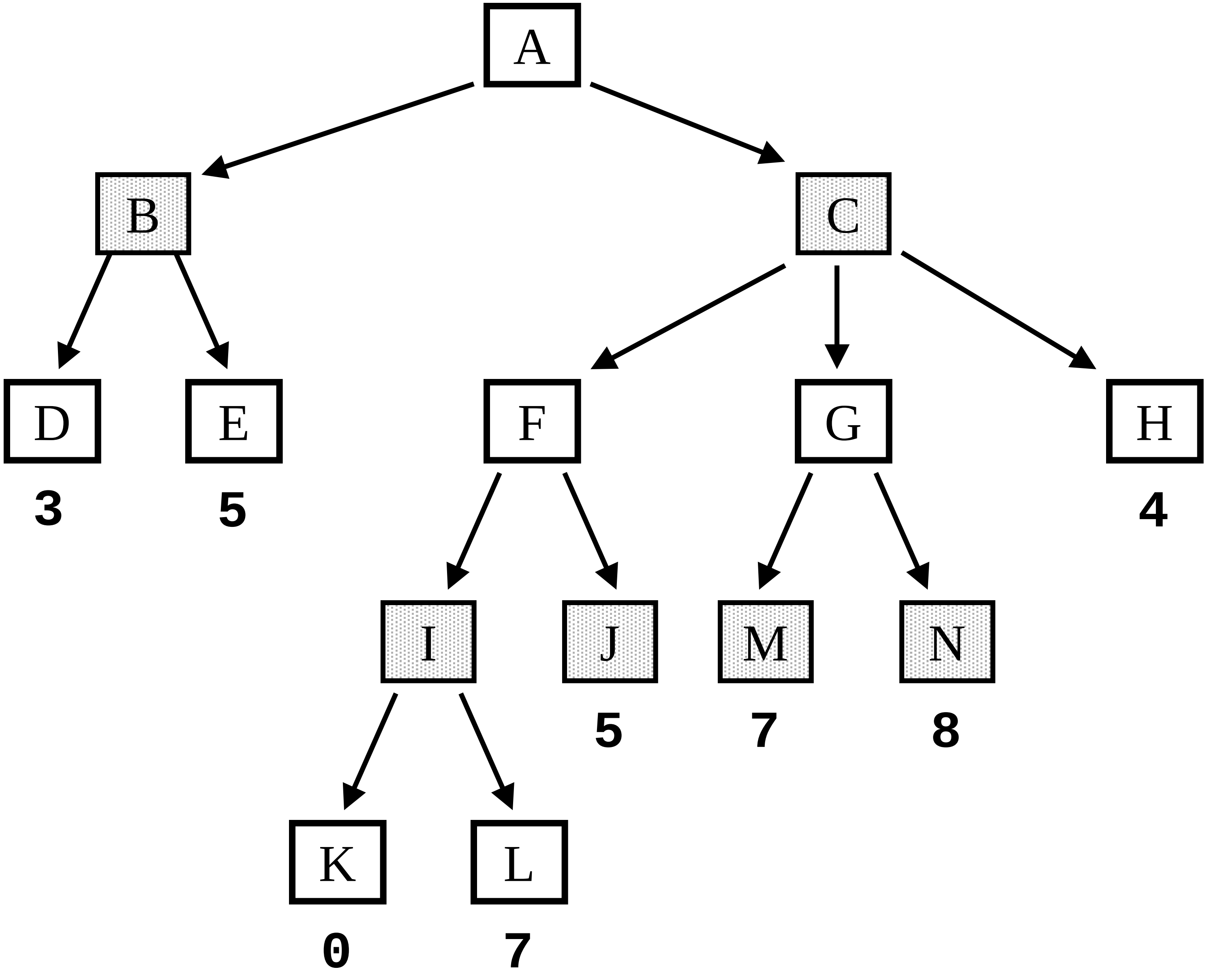
- At maximizing nodes, we stop evaluating children as soon as we get a child whose value is greater than the current upper bound (beta).

Alpha & Beta

- At the root of the search tree, alpha is set to -∞ and beta is set to +∞.
- Maximizing nodes update alpha from the values of children.
- Minimizing nodes update beta from the value of children.
- If alpha > beta, stop evaluating children.

Movement of Alpha and Beta

- Each node passes the current value of alpha and beta to each child node evaluated.
- Children nodes update their copy of alpha and beta, but do not pass alpha or beta back up the tree.
- Minimizing nodes return beta as the value of the node.
- Maximizing nodes return alpha as the value of the node.


A
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7

The Effectiveness of Alpha-Beta

- The effectiveness depends on the order in which children are visited.
- In the best case, the effective branching factor will be reduced from *b* to *sqrt(b)*.
- In an average case (random values of leaves) the branching factor is reduced to *b/logb*.

The Horizon Effect

- Using a fixed depth search can lead to the following:
  - A bad event is inevitable.
  - The event is postponed by selecting only those moves in which the event is not visible (it is over the horizon).
  - Extending the depth only moves the horizon, it doesn’t eliminate the problem.

Quiescence

- Using a fixed depth search can lead to other problems:
    - it's not fair to evaluate a board in the middle of an exchange of Chess pieces.
    - What if we choose an odd number for the search depth on the game of MinMax?
- The evaluation function should only be applied to states that are *quiescent* (relatively stable).

Pattern-Directed Play

- Encode a bunch of patterns and some information that indicates what move should be selected if the game state ever matches the pattern.
- *Book play:* often used in Chess programs for the beginning and ending of games.

Iterative Deepening

- Many games have time constraints.
- It is hard to estimate how long the search to a fixed depth will take (due to pruning).
- Ideally we would like to provide the best answer we can, knowing that time could run out at any point in the search.
- One solution is to evaluate the choices with increasing depths.

Iterative Deepening

- There is lots of repetition!
- The repeated computation is small compared to the new computation.
- Example: branching factor 10
  - depth 3: 1,000 leaf nodes
  - depth 4: 10,000 leaf nodes
  - depth 5: 100,000 leaf nodes

A* Iterative Deepening

- Iterative deepening can also be used with A*.

1. Set `THRESHOLD` to be *f*(start_state).
2. Depth-first search, don’t explore any nodes whose *f* value is greater than `THRESHOLD`.
3. If no solution is found, increase `THRESHOLD` and go back to step 2.