Discussion 2/18

Exercise 1:

Let n be an integer.

Show that

if $2n^2+n+9$ is odd, then n is even.

using

a) an indirect proof

b) a proof by contradiction

c) a direct proof.

This is a proof of an implication of the form $p \to q$

p: $2n^2+n+9$ is odd $\qquad \neg p$: $2n^2+n+9$ is even

q: n is even $\qquad \neg q$: n is odd.

a) Indirect proof

To show $p \to q$ we can show instead that $\neg q \to \neg p$.

I assume $\neg q$ is true, n is odd, there exists an integer k such that $n = 2k+1$

$$2n^2 + n + 9 = 2(2k+1)^2 + 2k+1+9$$
$$= 2(2k+1)^2 + 2k + 10$$
$$= 2\big[\underbrace{(2k+1)^2 + k + 5}_{\text{integer}}\big]$$

Therefore $2n^2 + n + 9$ is even: $\neg p$ is true.

b) Proof by contradiction.

We want to prove $(p \to q)$ is true.

We assume $\neg(p \to q)$ is true.

$$\neg(\neg p \vee q)$$

$p \wedge \neg q$ is true.

To prove $p \to q$ is true by contradiction, we assume p is true AND $\neg q$ is true.

- p is true: $2n^2 + n + 9$ is odd,
  There exists an integer k such that
  $$\boxed{2n^2 + n + 9 = 2k + 1}$$
- $\neg q$ is true, n is odd, there exists an integer l such that $\boxed{n = 2l + 1.}$

$$2(2l+1)^2 + 2l + 1 + 9 = 2k + 1$$

$$\underbrace{2\left[(2l+1)^2 + l + 5\right]}_{\text{even}} = \underbrace{2k+1}_{\text{odd}}$$

But this is not possible: I have reached a contradiction!

Therefore $p \to q$ is true.

c) Direct proof

I assume p is true.

$2n^2 + n + 9$ is odd, there exists an integer k such that

$$2n^2 + n + 9 = 2k + 1$$

$$n = 2k + 1 - 2n^2 - 9$$

$$= 2k - 2n^2 - 8$$

$$= 2\,[\underbrace{k - n^2 - 4}_{\text{integer}}]$$

n is even : q is true.

Conclusion: I have shown $p \to q$ using a direct proof, indirect proof and proof by contradiction

Exercise 2:

Let n be an integer.

Show that $n^2 + n + 9$ is odd.

$n =$	$n^2 + n + 9$	
1	11	odd
2	15	odd
3	21	odd
4	29	odd

So this is true.

This would be a proof by example, but this is not valid.

$n(n+1)$ is always even.

$n^2 + n$ is even: There exists an integer k such that $n^2 + n = 2k$.

$$n^2 + n + 9 = 2k + 9 = 2(k+4) + 1$$

Therefore $n^2 + n + 9$ is always odd.

Exercise 3

Let n be a natural number.

Prove or $\boxed{\text{disprove}}$ that $2^n + 1$ is prime.

n	$2^n + 1$	
1	3	prime
2	5	prime
3	$9 = 3 \times 3$	not prime.