ECS 20: Discrete Mathematics

Lecture Notes on the Sieve of Eratosthenes

The Sieve of Eratosthenes is a very ancient algorithm for making a list of the prime numbers less than some integer a. The algorithm starts by creating an array n containing the integers $1 \ldots a$, so that $n[i] = i$. Then it runs the following procedure on the array:

```
For (i = 2 to (a − 1) )
    If (n[i] ≠ ∅)
        j = 2
        while ( j n[i] < a )
            n[ j n[i] ] = ∅
            j = j + 1
```

Theorem 1 *After running the procedure above, the elements of* n *which are not* $\emptyset$ *are prime.*

Proof: First, we show that no prime number is set to $\emptyset$. A number k which is set to $\emptyset$ is of the form $j\ n[i]$, where $j, n[i] \in \mathbf{Z}^+$ (the set of integers greater than zero). So $j, n[i]$ are factors of k, and k is not prime.

We still need to show that every composite number k in the table is set to $\emptyset$. Since k is composite, it has some prime factor $p < k$. The integer p is never set to $\emptyset$, since it is prime, so all multiples of p in the tavle will be set to $\emptyset$ by the procedure. So k will be set to $\emptyset$.