

## Proof from Lecture on 1/4

**Theorem 1** *The memoized change-making algorithm calls the function `change()` at most  $5n$  times, where  $n$  is the input amount for which we want to make change.*

**Proof:** A *branching* call to the function `change(a)` gets the answer by doing the recursive calls to `change(a-25)`, `change(a-10)` etc. Any other call, answered either by using `memoDictionary` or by returning one or infinity, we shall refer to as a *memo* call. There are at most  $n$  branching calls, one for each  $1 < a \leq n$ . Each branching call makes at most four recursive calls. A memo call makes no recursive calls. So there are at most  $4n$  recursive calls to `change()`. This makes  $5n$  total, the  $n$  branching calls plus their  $4n$  recursive calls.