

**1. Finding a redundant edge**

Input: A connected, undirected graph  $G$ .

Output: An edge which can be removed from  $G = (V, E)$  while still leaving  $G$  connected, or `null` if no such edge exists.

Find an  $O(n)$  algorithm to solve this problem, where  $n = |V|$ . Recall that  $|E|$  can be  $\Omega(n^2)$ .

**2. Problem 17.2: Dynamic binary search****3. Disjoint sets**

a) Prove by induction that a node of rank  $k$  in a disjoint-set data structure must be the root of a tree containing at least  $2^k$  nodes.

b) Consider maintaining the disjoint-set data structure without path compression; that is, we do not change all the parent pointers along the search path to point to the root during a FIND operation. Prove that any  $m$  operations including at most  $n$  MAKE-SET operations requires time  $O(m \lg n)$ .

**4. Recurrence relation**

Get the best asymptotic upper bound you can for the recurrence relation:

$$T(n) = T(\lg n) + 1$$