1. Let A be nonsingular, and let $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n > 0$ be its singular values.

   (a) Find the SVD of A^{-1} in terms of the SVD of A. What are the singular values and singular vectors of A^{-1}?

   (b) Deduce that $\|A^{-1}\|_2 = \sigma_n^{-1}$ and $\text{cond}(A) = \|A\|_2\|A^{-1}\|_2 = \sigma_1/\sigma_n$, where $\text{cond}(A)$ is the condition number of A.

2. Ex 7.1

3. Ex 7.3

4. If $A = ab^T$, where $a \in \mathbb{R}^m$ and $b \in \mathbb{R}^n$, what is the first (largest) singular triplet (σ_1, u_1, v_1)?

5. (a) Let $A \in \mathbb{R}^{n\times n}$ be symmetric and positive definite. Determine the SVD of A from the eigenvalue decomposition of A.

   (b) Let $A \in \mathbb{R}^{n\times n}$ be symmetric and indefinite. Determine the SVD of A from the eigenvalue decomposition of A.

6. Let $A \in \mathbb{R}^{n\times n}$ of full rank. Use the SVD to determine a polar decomposition of A, i.e., $A = QP$ where Q is orthogonal, and $P = P^T > 0$.

   Note: (1) this is analogous to the polar form $z = re^{i\theta}$ of a complex scalar z, where $i = \sqrt{-1}$.
   (2) Inspired to learn more about the polar decomposition. Try the problems in Exercise 7.8.
   (3) The plar decomposition has wide applications, such as animation.

7. Ex. 7.10 ("Latent semantic analysis") — *option*