Block SAPOR: Block Second-Order Arnoldi Method for Passive Order Reduction of Multi-Input Multi-Output RCS Interconnect Circuits

Bang Liu[1], Xuan Zeng[1], Yangfeng Su[2], Jun Tao[1], Zhaojun Bai[3], Charles Chiang[4], Dian Zhou[1,5]

[1]ASIC & System State Key Lab., Microelectronics Dept., Fudan University, Shanghai, P.R. China
[2]Mathematics Dept., Fudan University, Shanghai, P.R.China
[3]Department of Computer Science, University of California Davis, CA 95616, USA
[4]Advanced Technology Group, Synopsys Inc., Mountain View, CA 94043, USA
[5]Electrical Engineering Dept., University of Texas at Dallas, Richardson, TX 75080, USA

Abstract - Recently model order reduction techniques for second-order systems have obtained many research interests for the simulation of RCS interconnect circuits employing susceptance elements. In this paper, we propose a Block SAPOR (Block Second-order Arnoldi method for Passive Order Reduction) for Multi-Input Multi-Output RCS Circuits. The proposed Block SAPOR algorithm can simultaneously guarantee passivity and achieve higher accuracy than the first order reduction technique PRIMA. Most importantly, the reduced system matrices obtained by the proposed method can preserve the structure of the original system matrices. Such a nice property makes it possible to construct an equivalent RCS circuit for the reduced system.

I. INTRODUCTION

In today's high-speed deep sub-micron ULSI design, interconnect has become a dominating factor in determining the performance and reliability of the whole chip. Generally, the on-chip interconnects are modeled as RLC circuits. As the operating frequencies keep increasing, magnetic coupling effects of interconnects become more and more prominent. Consequently, the resulted partial inductance matrix is extremely large and dense [1], which limits the application of fast and accuracy numerical techniques in interconnect simulation. As the inverse of a partial inductance matrix, susceptance has recently emerged as an alternative way for modeling magnetic coupling [2-4]. Since the mutual susceptance terms drop off much faster than the mutual inductance terms as the distance increases, the susceptance matrix is diagonally dominant and can be sparsified by simple truncation without lossing positive definiteness. This enables the development of fast simulation methods.

Model-order reduction (MOR) techniques have been well investigated during the last decade to fasten the interconnect simulation. Usually, a linear circuit can be equivalently formulated in the form of a first-order system [5][6], or a second-order system [4]. Hence MOR techniques can be classified into two categories, accordingly.

For MOR of the first-order formulation, the pioneering work is AWE [7], which uses a reduced-order system to match the explicitly-calculated moments of the original system. However, AWE suffers from numerical instability and cannot generate high-order models. Therefore, Krylov subspace based MOR techniques [6] were proposed later on. They often lead to a numerically stable order reduction process, which is highly desired for practical applications. Furthermore, special attention has been paid to maintain the passivity of the reduced-order model. In [5], PRIMA was developed based on Arnoldi process, which may provide guaranteed passivity. However, when directly applied to RCS circuits, PRIMA cannot guarantee passivity [4].

For an RCS circuit, it's better to be formulated as a second-order system, since many good properties of the susceptance matrix can be preserved in this form [4]. However, up to now, existing MOR techniques [9][4] for the second-order formulation leave many key issues to be resolved. For instance, ENOR [9] is not numerically stable while SMOR [4] cannot match the moments of the original system exactly. Therefore, an efficient Krylov subspace based MOR technique is much desired for the second-order formulation, just like PRIMA for the first-order systems.

Recently, Bai and Su introduced a second-order Arnoldi method (SOAR) [10] for the solution of the quadratic eigenvalue problem. Based on SOAR [10], an SAPOR (Second-order Arnoldi method for Passive Order Reduction) was further developed in [11] for SISO (Single-Input Single-Output) RCS circuits to guarantee the numerical stability, accuracy as well as passivity of the reduced system.

In this paper, we propose Block SAPOR: Block Second-order Arnoldi method for Passive Order Reduction of MIMO (Multi-Input Multi-Output) RCS circuits. This novel reduction method is numerically stable and passivity-guaranteed. Furthermore, it outperforms PRIMA

This research is supported partly by NSFC research project 90307017, 60176017, 90207002, partly by Synopsys Inc., partly by Cross-Century Outstanding Schoolar's fund of Ministry of Education of China, partly by National 863 plan projects 2004AA1Z1050, key project of Chinese Ministry of Education (No. 02095), Shanghai Science and Technology committee Key project 04JC14015, and partly by NSF grants CCR-0098275 and CCR-0306298.

with better accuracy for the same reduced order. Contrary to the existing MOR methods, which can not generate the equivelant circuits for the reduced systems, the Block SAPOR algorithm can construct the equivalent circuit for the reduced system.

The rest of the paper is organized as follows. In section II, a brief review of PRIMA is presented. In section III, the novel Block SAPOR algorithm is developed in detail. In section IV, a comparison of Block SAPOR and PRIMA is introduced. Numerical experiments are demonstrated in section V to show the efficiency of our proposed method. Concluding remarks are drawn in Section VI.

II. REVIEW OF PRIMA

A. Formulation of RLC circuits

For a *p*-input *q*-output MIMO RLC interconnect network, time-domain Modified Nodal Analysis (MNA) circuit equations can be given in the following.

$$C_X \dot{X}(t) + G_X X(t) = BU(t) \tag{1}$$

$$Y(t) = L^T X(t) \tag{2}$$

where $X \in R^{N+M}$ represents the unknown vector, which consists of N nodal voltages, denoted by $V \in R^N$ and M auxiliary branch currents, denoted by $I_b \in R^M$. The matrices $C_X \in R^{(N+M)\times(N+M)}$ and $G_X \in R^{(N+M)\times(N+M)}$ are the system matrices. $U \in R^p$ and $Y \in R^q$ are the input vector and output vector. $B \in R^{(N+M)\times p}$ and $L \in R^{q\times(N+M)}$ denote the incidence matrices for the input sources and output variables, respectively.

Generally, equation (1) is formulated as:

$$\begin{bmatrix} C & 0 \\ 0 & H \end{bmatrix} \begin{bmatrix} \dot{V}(t) \\ \dot{I}_b(t) \end{bmatrix} + \begin{bmatrix} G & E_L \\ -E_L^T & 0 \end{bmatrix} \begin{bmatrix} V(t) \\ I_b(t) \end{bmatrix} = BU(t) \tag{3}$$

where $C \in R^{N\times N}, H \in R^{M\times M}$ and $G \in R^{N\times N}$ represent the contributions of the capacitors, the inductances and the resistances, respectively. $E_L \in R^{N\times M}$ is the incidence matrix for inductances.

B. PRIMA

In frequency domain, equations (1) and (2) can be rewritten as (4) and (5), which is a first-order system, in terms of *s*.

$$(sC_X + G_X)X(s) = BU(s) \tag{4}$$

$$Y(s) = L^T X(s) \tag{5}$$

With unit impulse excitations at the inputs, we can easily obtain the system transfer function matrix.

$$H(s) = L^T (G_X + sC_X)^{-1} B \tag{6}$$

Apply Taylor expansion to *H(s)*, we have

$$H(s) = H_0 + H_1 s + H_2 s^2 + \cdots \tag{7}$$

where H_0, H_1, H_2, ... are the block moments of *H*. These block moments can be computed using the relation:

$$H_i = L^T A_X{}^i R \qquad \left(H_i \in \Re^{q\times p}\right) \tag{8}$$

where $A_X = -G_X{}^{-1} C_X$, $R = G_X{}^{-1} B$.

In PRIMA, a block Arnoldi procedure is employed for the construction of an orthonormal basis *W* spanning the block Krylov subspace as described below (for simplicity, we assume in the paper that the reduced order *n* is *k* times of *p*)

$$Kr(A_X, R) = span\left\{R, A_X R, A_X{}^2 R, \cdots A_X{}^{k-1} R\right\} \quad k = n/p \tag{9}$$

Then by performing an orthogonal projection on the original system using *W*, we reduce the system (1) and (2) of order $N+M$ to a system of order *n* as shown in (10) and (11).

$$(s\tilde{C}_X + \tilde{G}_X)\tilde{X}(s) = \tilde{B}U(s) \tag{10}$$

$$Y(s) = \tilde{L}^T \tilde{X}(s) \tag{11}$$

where $\tilde{C}_X = W^T C_X W$, $\tilde{G}_X = W^T G_X W$, $\tilde{X} = W^T X$, $\tilde{B} = W^T B$ and $\tilde{L} = W^T L$.

The transfer function matrix of the reduced system can be described as

$$\tilde{H}(s) = \tilde{L}^T (\tilde{G}_X + s\tilde{C}_X)^{-1} \tilde{B} \tag{12}$$

It is proven in [5] that *k* block moments of the transfer function matrix *H(s)* are preserved in the reduced system. Moreover, when applied to the system described in (3), PRIMA can generate a passive reduced-order system [5].

III. BLOCK SAPOR

A. Formulation of RCS circuits

For RCS circuits, since the susceptance *S* matrix can be regarded as the inverse of the inductance matrix *H*, the MNA equations (3) and (2) may be formulated as (13) and (14). Note that since only the nodal voltages are of interests in most cases, we assume the output variables are only relevant to nodal voltages. Moreover, for simplicity, we assume the inputs are all current sources.

$$\begin{bmatrix} G & E_S \\ -SE_S^T & 0 \end{bmatrix} \begin{bmatrix} V(t) \\ I_b(t) \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{V}(t) \\ \dot{I}_b(t) \end{bmatrix} = \begin{bmatrix} B_u U(t) \\ 0 \end{bmatrix} \tag{13}$$

$$Y(t) = \begin{bmatrix} L_V{}^T & 0 \end{bmatrix} \begin{bmatrix} V(t) \\ I_b(t) \end{bmatrix} = L_V{}^T V(t) \tag{14}$$

where $E_S = E_L$, $B_u \in R^{N\times p}$ and $L_V \in R^{q\times N}$ are incidence matrices for susceptances, current sources and the output voltage variables, respectively.

Performing the Laplace transform to (13) and (14), we have the MNA equations in frequency domain.

$$\left(\begin{bmatrix} G & E_S \\ -SE_S^T & 0 \end{bmatrix} + s \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \right) \begin{bmatrix} V(s) \\ I_b(s) \end{bmatrix} = \begin{bmatrix} B_u U(s) \\ 0 \end{bmatrix} \tag{15}$$

$$Y(s) = L_V{}^T V(s) \tag{16}$$

where $V(s), I_b(s)$ and $U(s)$ are Laplace transforms of $V(t), I_b(t)$ and $U(t)$, respectively. Obviously, this is also a first-order system, in terms of *s*.

In most applications, the auxiliary currents are generally intermediate variables. Therefore, we may eliminate $I_b(s)$ from (15). From the lower part of the above frequency domain equation (15), it can be obtained that

$$I_b(s) = \frac{1}{s} SE_S^T V(s) \tag{17}$$

Substituting (17) into the upper part of (15), we have the formula (18), which is a second-order formulation

equivalent to (15), except for the elimination of the auxiliary branch currents. Therefore, RCS circuit can be viewed as a second-order system

$$\left(sC+G+\frac{1}{s}\Gamma\right)V(s)=B_uU(s) \tag{18}$$

where $\Gamma=E_SSE_S^T$, and the matrices *C, G* and *Γ* are all symmetry positive semi-definite.

B. Block SAPOR

Similar to the output to input transfer function *H(s)* in (7), we define node voltage variables $V(s)$ to input variables $U(s)$ transfer function $H_V(s)$ in the following.

$$V(s)=H_V(s)U(s) \tag{19}$$

The entry in the *j*th row, *k*th column of $H_V(s)$ is the response of *j*th voltage variable under the excitation of the *k*th source.

According to (18), we have

$$\left(s^2C+sG+\Gamma\right)H_V(s)=sB_u \tag{20}$$

Shift (20) with $s=s_0+\sigma$, we have

$$\left(\sigma^2C+\sigma D+K\right)H_V(\sigma)=B_0+B_1\sigma \tag{21}$$

where $D=2s_0C+G$, $K=s_0^2C+s_0G+\Gamma$, $B_0=s_0B_u$, and $B_1=B_u$.

Apply Taylor expansion to $H_V(\sigma)$ in (21), we have

$$\left(\sigma^2C+\sigma D+K\right)\left(H_{V0}+H_{V1}\sigma+\cdots H_{Vi}\sigma^i+\cdots\right)=B_0+B_1\sigma \tag{22}$$

where $H_{V0},H_{V1},\cdots,H_{Vi},\cdots$ are defined as the block moments of H_V. In circuit terms, the entry in the *j*th row, *k*th column of H_{Vi} is the *i*th moments of the *j*th voltage variable under the excitation of the *k*th source . By comparing the left side and the right side of the above equation, we may have the following recurrence relation.

$$H_{V0}=K^{-1}B_0 \qquad H_{V1}=-K^{-1}DH_{V0}+K^{-1}B_1$$
$$H_{Vi}=-K^{-1}DH_{Vi-1}-K^{-1}CH_{Vi-2} \quad \text{for } i\geq 2 \tag{23}$$

Numerical instability will occur if we use the above recurrence to explicitly calculate the block moments of H_V. Instead, a Krylov subspace based technique is more desired to obtain an orthonormal basis of the block moment space of $H_V(s)$. In [10], a second-order Arnoldi method (SOAR) was proposed for the quadratic eigenvalue problem. The SOAR method is generalized and a Second-order Arnoldi method for Passive Order Reduction (SAPOR) of SISO RCS circuits is introduced in [11]. SAPOR is passivity guaranteed [11] and numerical experiments show the numerical stability of SAPOR. In the following, we will extend SAPOR to block SAPOR for MIMO RCS circuits.

B-1. System Linearization

Introduce a new matrix $H_Z(\sigma)$ satisfying

$$\sigma CH_V(\sigma)+H_Z(\sigma)=B_1 \tag{24}$$

Substituting (24) into (21), we may have

$$-\sigma H_Z(\sigma)+\sigma DH_V(\sigma)+KH_V(\sigma)=B_0 \tag{25}$$

Combining (24) and (25), we get

$$\left(I-\sigma A\right)\begin{bmatrix}H_V(\sigma)\\H_Z(\sigma)\end{bmatrix}=\begin{bmatrix}Q_0\\P_0\end{bmatrix} \tag{26}$$

Algorithm: Block SOAR

Input: A, Q_0, P_0 and integer orders *n, p*

Output: the orthonormal matrix Q

1. $k=\dfrac{n}{p}$
2. $\begin{bmatrix}Q_1\\P_1\end{bmatrix}=SOrth\left(\begin{bmatrix}Q_0\\P_0\end{bmatrix}\right)$
3. For $i=1,\ 2,\ \cdots,\ k-1$
4. $\quad\begin{bmatrix}\hat{Q}_{i+1}\\\hat{P}_{i+1}\end{bmatrix}=A\begin{bmatrix}Q_i\\P_i\end{bmatrix}$
5. $\quad$For $j=1,\ 2,\ \cdots,\ i$
6. $\quad\quad H_{ji}=Q_j^T\hat{Q}_{i+1}$
7. $\quad\quad\begin{bmatrix}\hat{Q}_{i+1}\\\hat{P}_{i+1}\end{bmatrix}=\begin{bmatrix}\hat{Q}_{i+1}\\\hat{P}_{i+1}\end{bmatrix}-\begin{bmatrix}Q_j\\P_j\end{bmatrix}H_{ji}$
8. $\quad$End
9. $\quad\begin{bmatrix}Q_{i+1}\\P_{i+1}\end{bmatrix}=SOrth\left(\begin{bmatrix}\hat{Q}_{i+1}\\\hat{P}_{i+1}\end{bmatrix}\right)$
10. End
11. $Q=\begin{bmatrix}Q_1 & \cdots & Q_k\end{bmatrix}$

Figure 1. Block SOAR algorithm

where $A=\begin{bmatrix}-K^{-1}D & K^{-1}\\ -C & 0\end{bmatrix}$, $Q_0=K^{-1}B_0$ and $P_0=B_1$.

By moving $(I-\sigma A)$ to the RHS of (26) and performing a Maclaurin series expansion, we have:

$$\begin{bmatrix}H_V(\sigma)\\H_Z(\sigma)\end{bmatrix}=\left(I+\sigma A+\sigma^2A^2+\sigma^3A^3+\cdots\right)\begin{bmatrix}Q_0\\P_0\end{bmatrix} \tag{27}$$

Here, $A^i\begin{bmatrix}Q_0\\P_0\end{bmatrix}$ is the *i*-th block moment of $\begin{bmatrix}H_V(\sigma)\\H_Z(\sigma)\end{bmatrix}$, and $\begin{bmatrix}I & 0\end{bmatrix}A^i\begin{bmatrix}Q_0\\P_0\end{bmatrix}$ must be equal to the *i*-th block moment of $H_V(\sigma)$. Moreover, Q_0 and P_0 are actually the zeroth block moments of H_V and H_Z, respectively.

B-2. Orthonormalization Process

To generate an orthonormal basis of the block second-order Krylov subspace spanned by the block moments of H_V, we propose the block SOAR procedure. Block SOAR is the block extension of SOAR [11], similar to the Block Arnoldi being an extension of Arnoldi.

The Block SOAR algorithm is summarized in Figure 1. In order to generate an orthonormal basis Q of the block moment space of H_V, we need to match the first *k* block moments of H_V. Therefore, *k* iterations are involved in

Algorithm: SOrth

Input: $\hat{Q}_m, \hat{P}_m$, p

Output: the orthonormal matrices Q_m, P_m

1. $k = \frac{n}{p}$
2. set $\hat{Q}_m = \begin{bmatrix} \hat{q}_1 & \hat{q}_2 & \cdots & \hat{q}_p \end{bmatrix}$ $\hat{P}_m = \begin{bmatrix} \hat{p}_1 & \hat{p}_2 & \cdots & \hat{p}_p \end{bmatrix}$
3. For $i = 1, 2, \cdots, p$
4. $\begin{bmatrix} q_i \\ p_i \end{bmatrix} = \begin{bmatrix} \hat{q}_i \\ \hat{p}_i \end{bmatrix}$
5. For $j = 1, 2, \cdots, i-1$
6. $R_{ji} = q_j^{'} q_i$
7. $\begin{bmatrix} q_i \\ p_i \end{bmatrix} = \begin{bmatrix} q_i \\ p_i \end{bmatrix} - R_{ji} \cdot \begin{bmatrix} q_j \\ p_j \end{bmatrix}$
8. End
9. $R_{ii} = \|q_i\|$
10. If $R_{ii} \cong 0$, stop (deflation)
11. $\begin{bmatrix} q_i \\ p_i \end{bmatrix} = \frac{1}{R_{ii}} \begin{bmatrix} q_i \\ p_i \end{bmatrix}$
12. End
13. End
14. $Q_m = \begin{bmatrix} q_1 & q_2 & \cdots & q_p \end{bmatrix}$ $P_m = \begin{bmatrix} p_1 & p_2 & \cdots & p_p \end{bmatrix}$

Figure 2. SOrth Procedure

block SOAR procedure. In each iteration, a new $\begin{bmatrix} Q_i \\ P_i \end{bmatrix}$ is generated. Since a number of *p* basis vectors are included in each $\begin{bmatrix} Q_i \\ P_i \end{bmatrix}$, we need an extra orthonormalization procedure to realize the orthogonalization of these *p* basis vectors at the end of each iteration. We use Gram-Schmidt algorithm to realize the orthonormalization, which is called SOrth (Second-order Orthonormalization) procedure, as shown in Figure 2.

Once the orthonormal basis *Q* is obtained, we perform an orthogonal projection on the original second-order system (18) using *Q*, and obtain a reduced-order system of the same form.

$$\left(s\tilde{C} + \tilde{G} + \frac{1}{s}\tilde{\Gamma}\right)\tilde{V}(s) = \tilde{B}_u U(s) \tag{28}$$

$$Y(s) = \tilde{L}_V{}^T \tilde{V}(s) \tag{29}$$

where $\tilde{C} = Q^T C Q$, $\tilde{G} = Q^T G Q$, $\tilde{\Gamma} = Q^T \Gamma Q$, $\tilde{V} = Q^T V$, $\tilde{B}_u = Q^T B_u$ and $\tilde{L_V} = Q^T L_V$.

Since the matrices *C, G* and *Γ* in (18) are all symmetry positive semi-definite, it is proven in [9] that the orthogonal projection preserves the passivity of the original system. Similarly, we conclude that the reduced-order system in (28) has guaranteed passivity.

B-3. Summary of Block SAPOR

Our novel technique Block SAPOR (Second-Order Arnoldi based Passive Order Reduction) can be outlined as follows:

1) Formulating the RCS circuit as the second-order system in (18) and (16).
2) Shifting (18) with $s = s_0 + \sigma$ and obtain (21) and (26).
3) Using the block SOAR algorithm in Figure 1 and Sorth algorithm in Figure 2 to construct the orthonormal matrix *Q*.
4) Performing an orthogonal projection on the original system and obtain the reduced-order system as in (28) and (29).

In the following, we provide some theorems for Block SAPOR. For SISO cases, similar theorems have been proven in [12], and we can easily extend them to MIMO cases. Due to the limited space, the proofs for these theorems are omitted here.

Theorem 1: Under the assumption of no deflation and breakdown, the vectors $Q_1, Q_1, \cdots Q_k$ generated in Block SOAR algorithm form an orthonormal basis of the block moment space of H_V.

Theorem 2: If we use the matrix Q obtained by the Block SOAR algorithm to perform a projection on the system (18) and obtain the projected system (28), the reduced-order system (28) will match $\lfloor n/p \rfloor$ block moments of the output to input transfer function matrix of the original system (18).

Theorem 3: If $L_V = B_u$ in the original system (13) and (14), the first $2\lfloor n/p \rfloor$ block moments of the ouput to input transfer function matrix of the original system are matched in the reduced system by Block SAPOR.

IV. COMPARISON BETWEEN BLOCK SAPOR AND PRIMA

In this section, we compare the Block SAPOR algorithm with PRIMA in the following aspects.

A. Structure Preservation

In PRIMA, the original system matrices C_X *and* G_X have the block structure in (3), where *C, G* and *H* present the contribution of capacitances, resistances and inductances. After order reduction, the reduced system matrices $\tilde{C}_X$ *and* $\tilde{G}_X$ become dense, and no longer preserve the block structure of the original C_X *and* G_X in (3). Therefore, it is impossible to reconstruct an equivalent RLC circuit of *n-th* order reduced system.

However, Block SAPOR can preserve the structure of the orginal system. As formulated in (15), the original system with order of $N + M$ can also be formulated as (30) by introducing a new variable vector $\hat{I}$ satisfying $\hat{I} = E_S I_b$.

$$\left(\begin{bmatrix} G & I \\ -\Gamma & 0 \end{bmatrix} + s\begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}\right)\begin{bmatrix} V(s) \\ \hat{I}(s) \end{bmatrix} = \begin{bmatrix} B_u U(s) \\ 0 \end{bmatrix} \tag{30}$$

Using Block SAPOR, we obtain the reduced system (28)

with order of *n*. By defining a new variable vector $\tilde{I} = Q^T \hat{I}$, the reduced system (28) can be reformulated in (31).

$$\left(\begin{bmatrix} \widetilde{G} & I \\ -\widetilde{\Gamma} & 0 \end{bmatrix} + s \begin{bmatrix} \widetilde{C} & 0 \\ 0 & I \end{bmatrix} \right) \begin{bmatrix} \widetilde{V}(s) \\ \tilde{I}(s) \end{bmatrix} = \begin{bmatrix} \widetilde{B}_u U(s) \\ 0 \end{bmatrix} \quad (31)$$

Obviously, the above reduced-order system (31) exactly preserves the structure of the original system (30).

Based on the advantage of structure preservation, we can easily reconstruct an equivalent RCS circuits of *n*-th order to substitute the original large one of $N+M$ order, which may benefit the future simulation procedures.

We recall the reduced system (28), which is equivalent to (31), to clarify the basic idea of the equivalent circuit reconstruction. The equation (28) can be viewed as an expression of Kirchhoff Current Law, where the left hand part denotes the current contribution from the capacitances, resistances and susceptances in the equivalent circuit, and the right hand part denotesthe contribution of the current sources. For each node of the total *n* nodes in the equivalent circuit, we can easily acquire the capacitances, resistances, susceptances and current sources ,which are connected to the node,from the matrices $\widetilde{C}, \widetilde{G}, \widetilde{\Gamma}$ *and* $\widetilde{B}_u$, respectively.

B. *Superior Efficiency*

In PRIMA, the *n*-th order reduced system can be obtained in (32) by projecting the original system using an *n* dimensional projection subspace spanned by the columns of matrix *W* of $\Re^{(N+M)\times n}$.

$$W^T (sC_X + G_X) W W^T X(s) = W^T B U(s) \quad (32)$$

Similarly, in Block SAPOR, the reduced second-order system (28) with order of *n* is equivalent to a first-order system (33) with order of *n*+*M*.

$$\begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix} \left(\begin{bmatrix} G & E_S \\ -SE_S^T & 0 \end{bmatrix} + s \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \right) \begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix} \begin{bmatrix} V(s) \\ I_b(s) \end{bmatrix} = \begin{pmatrix} Q^T & 0 \\ 0 & I \end{pmatrix} \begin{bmatrix} B_u U(s) \\ 0 \end{bmatrix} \quad (33)$$

Equation (33) indicates that the reduced system by Block SAPOR is a projection of the original system in (15) by the projection matrix $\begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}$ of $\Re^{(N+M)\times(n+M)}$

Theorem 4 The *n* dimensional subspace spanned by the columns of *W* in (32) is included in the *n*+*M* dimensional subspace spanned by the columns of $\begin{pmatrix} Q & 0 \\ 0 & I \end{pmatrix}$ in (33).

Theorem 4 can be readily proved. From it, we can see that the projection subspace in Block SAPOR is larger than the one in PRIMA. As a result, Block SAPOR surely provides better accuracy than PRIMA for the same reduction order.

Furthermore, as described in theorem 3, when applied to those circuits where $L_V = B_u$, Block SAPOR can always ensure matching 2*k* block moments of the system transfer function matrix and guarantee passivity simultaneously.

However, PRIMA cannot ensure these two properties at the same time [6], i.e. if passivity is guaranteed, only *k* block moments can be matched, comparatively, if we prefer *2k* block moment matching, the passivity can no longer be preserved. Consequently, Block SAPOR can provide more prominent superiority to PRIMA in terms of accuracy and passivity.

C. *Lower Cost in Time and in Memory*

In PRIMA, RLC model is used, which results in the first-order system in (4) and (5). The introduction of extra current variables makes the system matrices are non-positive definite, which limits many efficient numerical techniques to be applied to MOR procedure. Comparatively, we use the RCS model instead of RLC model in Block SAPOR. The elimination of the intermediate current variables makes the matrices in the resulted second-order system (18) all symmetric positive definite and sparse. These merits of system matrices benefit time complexity in MOR computation. For example, a sparse Cholesky factorization of *K* can be employed to the matrix-vector multiplication involving K^{-1}.

To reduce memory consumption, it is possible to derive a memory saving version of Block SAPOR, which will save almost half memory usage comparing with PRIMA. The basic idea is similar to the SOAR procedure with memory saving introduced in [10], and we don't expand it here due to the limited length of this paper.

V. NUMERICAL EXPERIMENTS

In this section, we present numerical experiments to demonstrate the efficiency of the proposed Block SAPOR method. We will compare Block SAPOR with PRIMA. Both methods are implemented in MATLAB.

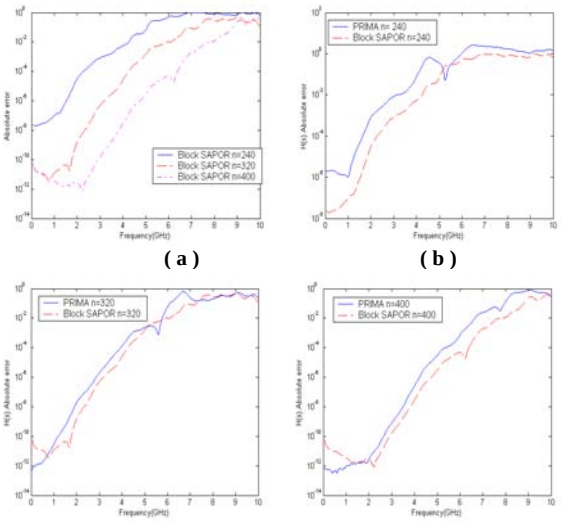
A. *Example I with* $L_V \neq B_u$

The first example is a 16-bit bus line circuit. Here the circuit is driven by 16 current sources at the near ends and observed at the output voltages at the far ends, which means $L_V \neq B_u$ in the corresponding MNA equations. The order of the circuit is 1746, including 1170 nodal voltages and 576 auxiliary currents.

For simplicity, we use the first element in transfer function matrix, which denotes the impulse response of the far end of the first line when only the source of the first line is active, as a criteria for judging the reduction accuracy. In Figure 3 (a), we plot the frequency response errors of Block SAPOR with three different reduced orders, i.e. 240,320 and 400. It can be seen that the reduced errors become smaller as the reduced order goes higher.

For comparison, we perform PRIMA to the same circuit. The comparisons of the frequency response errors by PRIMA and Block SAPOR are plotted in figure 3 (b,c,d), with the same reduced order 240, 320 and 400, respectively. We can see that Block SAPOR obtains better accuracy in a wider frequency range than PRIMA, which is owing to its larger projection space.

We also compare the run time between PRIMA and Block SAPOR in Table 1. The first part lists the time used for obtaining the reduced system. It is clear that the reduction time by Block SAPOR is much less than that by PRIMA. The second part of the table lists the total time, which includes the reduction time and the time for calculating the frequency responses of the reduced system. It can be seen



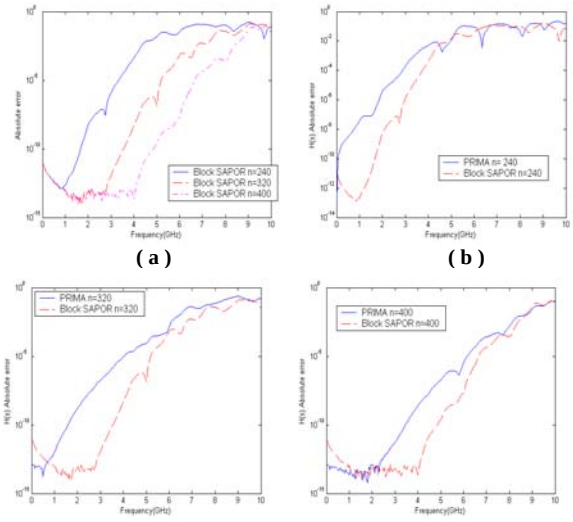
(a) (b)

(c) (d)

Figure 3. Comparison of frequency response errors in example I. (a) Block SAPOR with three different reduced order; (b,c,d) PRIMA and Block SAPOR with the same reduced-order, 240, 320 and 400, respectively

Table 1 Comparisons of run time in example I

Reduced order	Reduction time (s)		Total time (s)	
	PRIMA	Block SAPOR	PRIMA	Block SAPOR
240	21.141	5.265	22.688	11.172
320	24.562	7.422	27.609	20.079
400	29.235	9.812	34.407	32.406



(a) (b)

(c) (d)

Figure 4. Comparison of frequency response errors in example II. (a) Block SAPOR with three different reduced order; (b,c,d) PRIMA and Block SAPOR with the same reduced-order, 240, 320 and 400, respectively

that the total time speedup by Block SAPOR compared to PRIMA is not so significant as the reduction time speedup. This is because when calculating the reduced system by Block SAPOR, we didn't apply any special numerical technique. By taking advantage of the symmetry and positive definition properties in (28), simulation time for solving the reduced system by Block SAPOR can be further reduced.

B. *Example II with* $L_V = B_u$

In the second example, the same 16-bit bus line circuit is excited by 16 current sources at the near ends and the 16 voltage outputs at the same ends are observed. In this testing, we have $L_V = B_u$ in the corresponding MNA equations.

In Figure 4(a), we plot the frequency response errors of Block SAPOR with three different reduced orders, i.e. 240, 320 and 400. The comparisons of the frequency response errors by PRIMA and Block SAPOR are plotted in figure 4 (b-d), with the same reduced order 240, 320 and 400, respectively. We can see that when $L_V = B_u$, Block SAPOR is even more accurate than PRIMA comparing with the case of $L_V \neq B_u$. The run time for example II is similar to example I.

VI CONCLUSION

In this paper, we presented Block SAPOR algorithm for the model order reduction of MIMO RCS interconnect circuits. The novel technique can guarantee passivity and present higher accuracy than the well known PRIMA. Moreover, the reduced system matrices can preserve the structure of the original system matrices such that equivelent circuit can be developped for the reduced system.

Reference:

[1] D. Ling and A. Ruehli, Circuit Analysis, *Simulation and Design – Advances in CAD for VLSI. Vol. 3, Part II, Chap. 11, Elsevier Science Publisher,* 1987.

[2] A. Devgan, H. Ji and W. Dai, How to Efficiently Capture On-Chip Inductance Effects: Introducing a New Circuit Element K. *Proc. of IEEE/ACM ICCAD 2000*, pp. 150–155, 2000.

[3] H. Zheng, B. Krauter, M. Beattie and L. Pileggi, Window-Based Susceptance Models for Large-Scale RLC Circuit Analyses. *Proc. of IEEE/ACM DATE 2002*, pp. 628–633, 2002.

[4] H. Zheng and L. Pileggi, Robust and Passive Model Order Reduction for Circuits Containing Susceptance Elements. *Proc. of IEEE/ACM ICCAD 2002*, pp. 761–766, 2002.

[5] A. Odabasioglu, M. Celik and L. Pileggi, PRIMA: Passive Reduced-Order Interconnect Macromodeling Algorithm. *IEEE Trans. on CAD of Integrated Circuits and Systems*, vol. 17, no. 8, pp. 645–654, Aug. 1998.

[6] R. W. Freund, Reduced-Order Modeling Techniques Based on Krylov Subspaces and Their Use in Circuit Simulation. *Numerical Analysis Manuscript*, No. 98-3-02, Bell Laboratories, Feb. 1998.

[7] L. Pillage, and R. A. Rohrer, Asymptotic Waveform Evaluation for Timing Analysis. *IEEE Trans. on CAD of Integrated Circuits and Systems*, Vol. 9, No. 4, pp. 352–366, Apr. 1990.

[8] L. M. Silveira, M. Kamon, I. M. Elfadel, and J. White. Coordinate-transformed Arnoldi for generating guaranteed stable reduced-order models for RLC circuits.*Proc. Of ICCAD'96*, pages 288-294, San Jose, CA, November 1996.

[9] B. N. Sheehan, ENOR: Model Order Reduction of RLC Circuits Using Nodal Equations for Efficient Factorization. *Proc. of IEEE/ACM DAC '99*, pp. 17–21, 1999.

[10] Z. Bai and Y. –F. Su, SOAR: A Second-Order Arnoldi Method for the Solution of the Quadratic Eigenvalue Problem. *Computer Science Technical Report,* CSE-2003-21, University of California, Davis, 2003. To appear in *SIAM J. Matrix Anal. Appl.*

[11] Y. –F. Su, J, Wang, X, Zeng, Z Bai, Charles Chiang and D. Zhou, SAPOR: Second-Order Arnoldi Method for Passive Order Reduction of RCS Circuits. *Proc. of IEEE/ACM ICCAD 2004.*

[12] Z. Bai and Y. –F. Su, Dimension Reduction of Second-Order Dynamical Systems via a Second-Order Arnoldi Method. *Computer Science Technical Report*, CSE-2004-1, University of California, Davis, 2003. To appear in *SIAM J. Sci. Comp.*