Homework 2 – ECS 289, Winter 2016

1. **Composition.** Recall that the following CRC with input species X_1, X_2 and output species Y stably computes $y = \max(x_1, x_2)$

$$\begin{aligned} X_1 &\rightarrow Y + W_1 \\ X_2 &\rightarrow Y + W_2 \\ W_1 + W_2 &\rightarrow K \\ K + Y &\rightarrow \varnothing \end{aligned}$$

And the following CRC with input species Y, X_3 and output species Z stably computes $z = \min(y, x_3)$

$$Y + X_3 \rightarrow Z$$

   (a) Show that the CRC obtained by simply combining the above 5 reactions, with input species X_1, X_2, X_3 and output species Z, does *not* stably compute $z = \min(\max(x_1, x_2), x_3)$.

   (b) Design a CRC that stably computes $z = \min(\max(x_1, x_2), x_3)$.

2. **Combining function and predicate computation.** Design a CRC that stably computes the function

$$f(x_1, x_2) = \begin{cases} x_1, & \text{if } x_1 \geq x_2; \\ 0, & \text{otherwise.} \end{cases}$$

3. **Leader election.** Consider the following CRC, which stably computes $f(x) = 1$

$$\begin{aligned} X &\rightarrow Y \\ 2Y &\rightarrow Y \end{aligned}$$

   (a) Design a leaderless CRC that stably computes $f(x) = 2$, in which every reaction has at most two reactants and two products.

   (b) Describe how to generalize the previous CRC to produce, for each $k \in \mathbb{N}$, a leaderless CRC that stably computes the function $f(x) = k$, in which every reaction has at most two reactants and two products.