HW 1 Solution to the RNA matching count problem.

Problem 3. Now give recurrences that can be used to correctly count the number of permitted matchings, and give a DP that computes the count in $O(n^3)$ time. This may be a bit challenging.

Solution: $S(i, j)$ is defined as the number of permitted matchings involving the positions from i to j inclusive. It includes the empty matching as one of the matchings. For technical reasons, define $S(j, j) = S(j + 1, j) = 1$.

Then $S(i, j) = S(i + 1, j - 1)$

$+ \sum_{i<k\leq j:\texttt{i and k can match}}[S(i + 1, k - 1) \times S(k + 1, j)]$

$+ \sum_{i<k<j:\texttt{j and k can match}}[S(i + 1, k - 1) \times S(k + 1, j - 1)]$

The first term covers the case that neither i nor j are involved in a match. The first sum counts all of the cases where i is involved in a match to some position between $i + 1$ and j. Even though i is definately matched to a position k, j may or may not be involved in a match, and the number of those posibilities is $S(k + 1, j)$. The second sum counts the cases where j is definately involved in a match and i definately is not involved. We need to verify that all possible matchings are counted, and that none are double counted. The cases are: 1) that neither i nor j are involved in a match - that is covered in the first term, but not in either of the sums, since they count only matchings where i or j or both are in a match.

2) i definately is in a match. Those matchings are counted in the first sum, but not in the second sum or the first term.

3) i and j are both in a match. Those matchings are counted in the first sum, but not in the second sum or the first term.

4) j is in a match, but i is not. Those matchings are counted in the second sum.