

Buneman Graph

Exercise

1. We will outline the proof of the following theorem.

Theorem. The Buneman Graph $\mathcal{B}(H)$, for binary matrix H , is a median graph.

To prove the Theorem, first establish the following two facts:

- Show that $\mathcal{B}(H)$ is connected and that the length of the shortest path between two vertices $u, v \in \mathcal{B}(H)$ is equal to the number of components on which u and v differ. Actually, that part was proven in the book in the proof that a Buneman graph is a splits network, so you don't need to do that.
- Let v_1, v_2 , and v_3 be any three vertices of $\mathcal{B}(H)$. Consider vertex v such that the i th component of v is the block that agrees with the i th component of at least two of v_1, v_2 , and v_3 . Show that v is the unique median of v_1, v_2 , and v_3 .

Now complete the proof that the Buneman Graph, given H , is a median graph.