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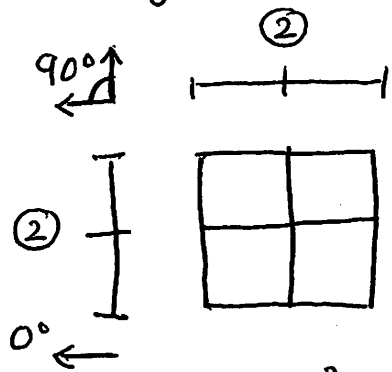
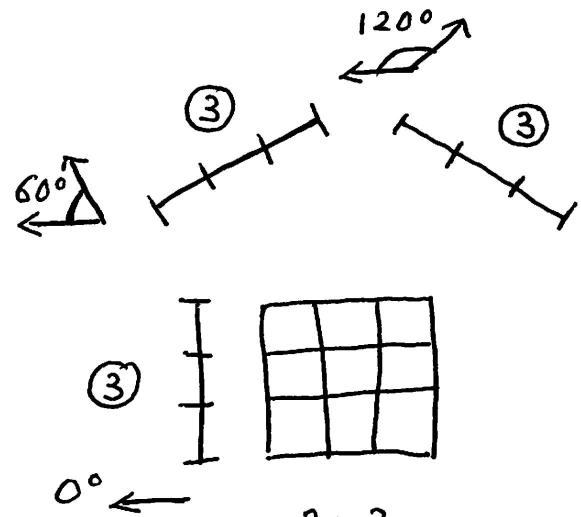
Thoughts Concerning an (Alternative?) Method for 3D Volume Image Data RECONSTRUCTION

1) On Resolution

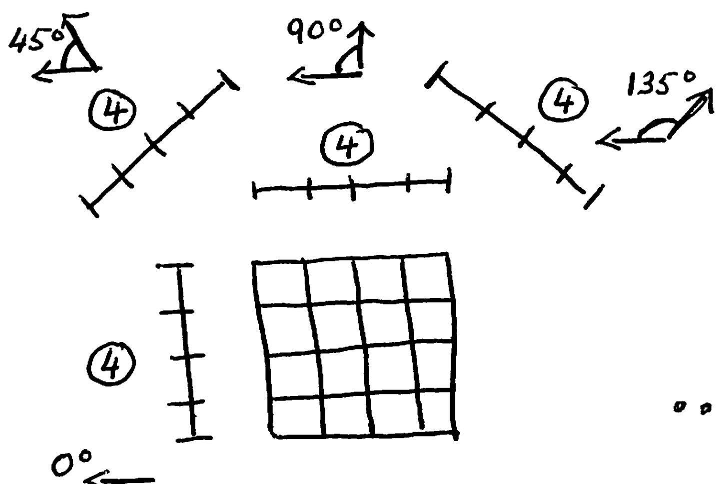
[Sinogram Setting]



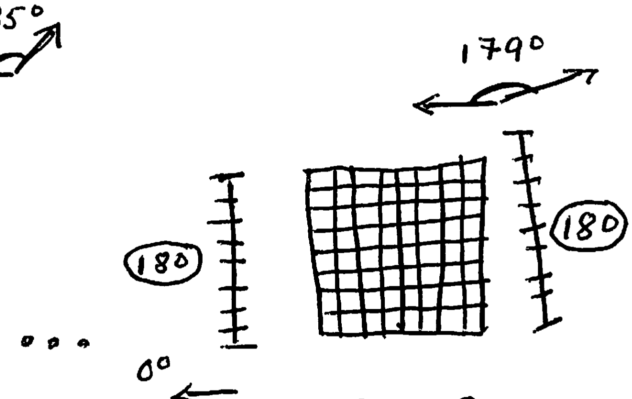
2 × 2



3 × 3



4 × 4



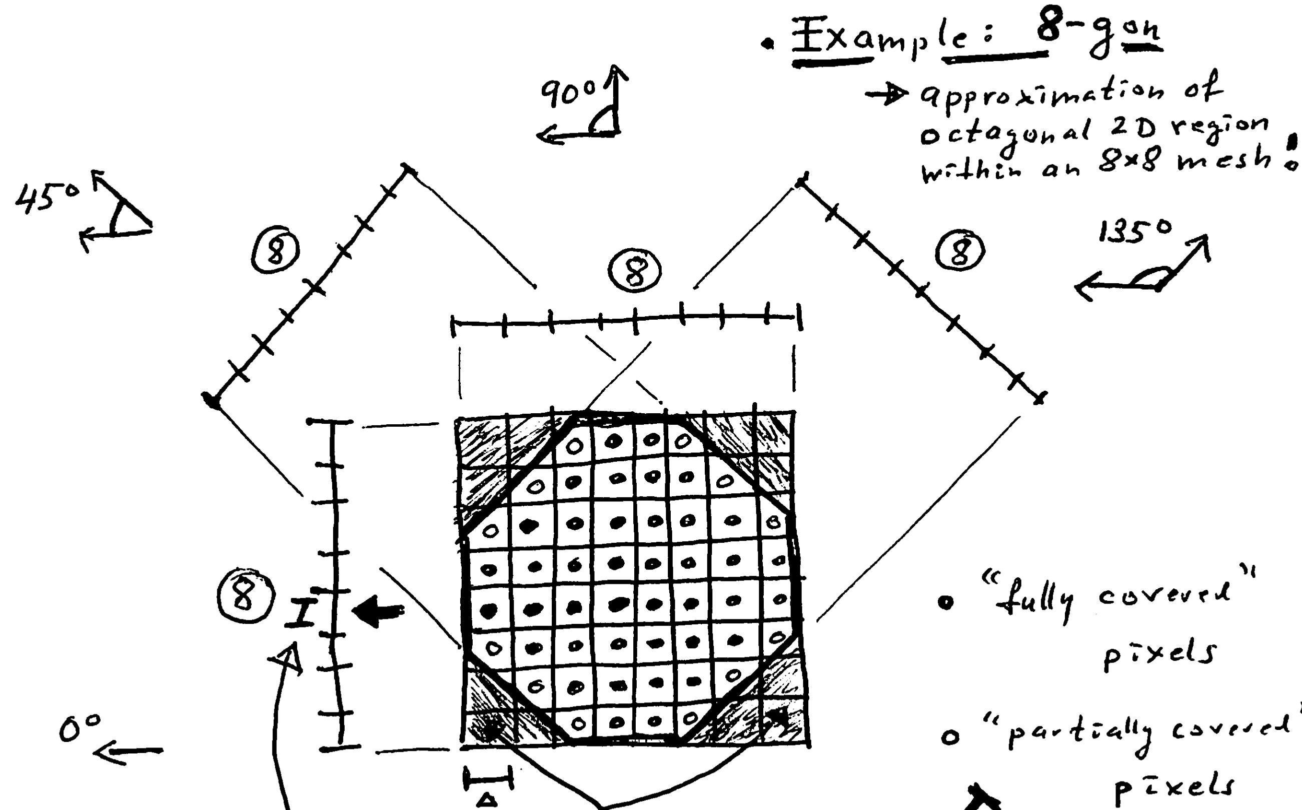
180 × 180

⇒ 2D Planar Image Data Example:

- Each 1D projection has N intervals.
- The N angles used for projection are $i \cdot \frac{180°}{N}$, $i = 0 \ldots (N-1)$.
- The 2D Planar Image has resolution $N \times N$.

RECONSTRUCTION – Cont'd.

2) On the 'n-gon Approximation' of a Rotating Object



– here: 8×8 measured I-values; < 8×8 i-values unknown inside octagonal region

Radon transform for intensity I:

$$\boxed{I = \sum_{j=0}^{7} \Delta \cdot i_j = \Delta \sum_{j=0}^{7} i_j,}$$

where i_j =

i_7							i_0

= one row of pixels.

– One obtains such a Radon sum for every measured value I.

given: set of I-values for all projections

unknowns: 2D pixel intensity values i of the 2D image

– WHAT IS BEST WAY TO WRITE/ ORGANIZE THESE I-VALUE EXPRESSIONS TO GET "IDEAL MATRIX" (of linear system)?

RECONSTRUCTION - Cont'd.

3) On Reconstruction Order / Smoothness

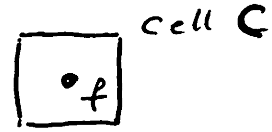
→ Use 2D $N \times N$ (image) grid to define, for example,

(i) a piecewise constant expansion/approx. of image

OR

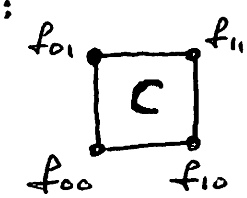
(ii) a piecewise bilinear expansion/approx. of image.

(i) cell-centric view:



$\Rightarrow F(C) = f$

(ii) vertex-centric view:



$\Rightarrow F(C) =$ "Bilinear" $(f_{00}, f_{10}, f_{01}, f_{11})$

→ Practical considerations / simplicity / robustness:

Use PIECEWISE CONSTANT reconstruction (using a relatively higher resolution)!

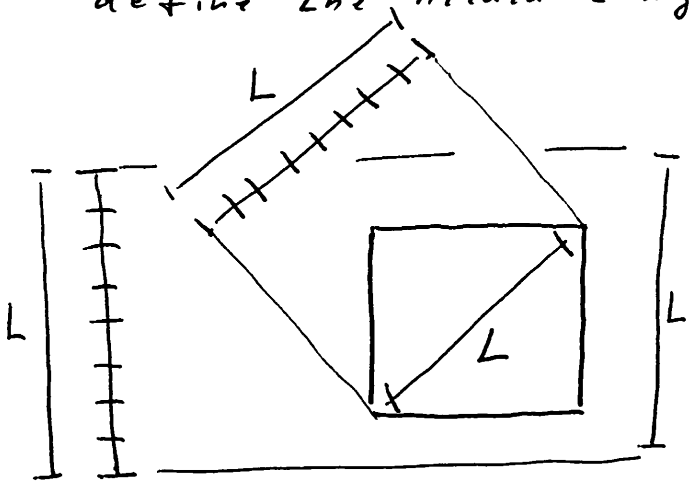
→ Related reference:

Lee Alan Westover, SPLATTING: A parallel, feed-forward volume rendering algorithm; 1991.

▣ RECONSTRUCTION – Cont'd.

4) On Defining the Linear System etc.

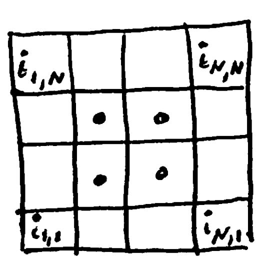
→ To reconstruct $N \times N$ pixel values over a square 2D domain, use the square's diagonal length to define the needed length for each 1D projection:



⇒ All 1D projection lines are of sufficient length to always "cover" the entire 2D image domain!

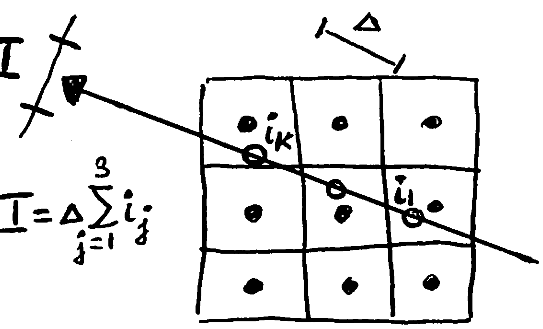
→ The linear system for the CELL CENTER values:

- the unknown i-values:



$N \times N$ unknown cell-center values $i_{j,k}$ ("∘")

- samples along a ray, "∘", do NOT coincide with "•" locations:

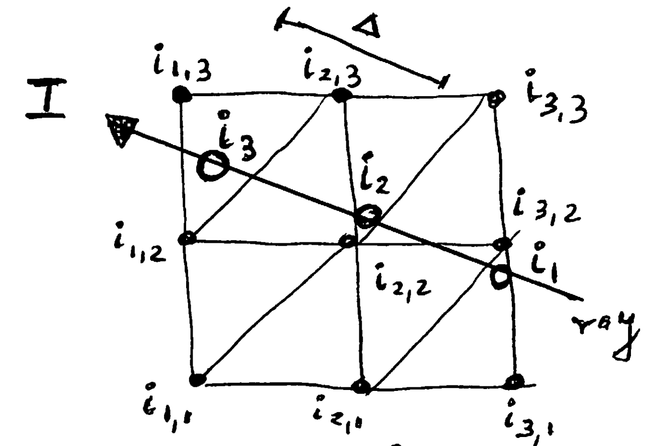


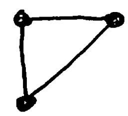
$$I = \Delta \sum_{j=1}^{3} i_j$$

⇒ Can define a Delaunay triangulation for the "•" points & express "∘" location via barycentric coords.!

■ RECONSTRUCTION – Cont'd.

4) The Linear system ...



 = Delaunay triangle in triangulation of cell centers "∘"

∘ sample location along ray, contributing to value I

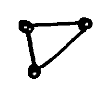
$$\Rightarrow I = \Delta \sum_{j=1}^{3} i_j$$

$$= \Delta \left(i_1 + i_2 + i_3 \right)$$

$$= \Delta \big(\alpha_{2,1}\, i_{2,1} + \alpha_{3,1}\, i_{3,1} + \alpha_{3,2}\, i_{3,2}$$

$$+ \alpha_{22}\, i_{2,2} + \alpha_{3,3}\, i_{3,3} + \alpha_{2,3}\, i_{2,3}$$

$$+ \alpha_{1,2}\, i_{1,2} + \alpha_{2,3}\, i_{2,3} + \alpha_{1,3}\, i_{1,3} \big) ,$$

where the α-values are the barycentric coordinates of a site/location "∘" relative to the Delaunay triangle  that contains "∘".

→ Parameter set-up should ideally lead to an overdetermined system for the unknown i-values, to be solved via the least-squares approach!

→ Resulting system to be "organized" to yield optimal EFFICIENCY!