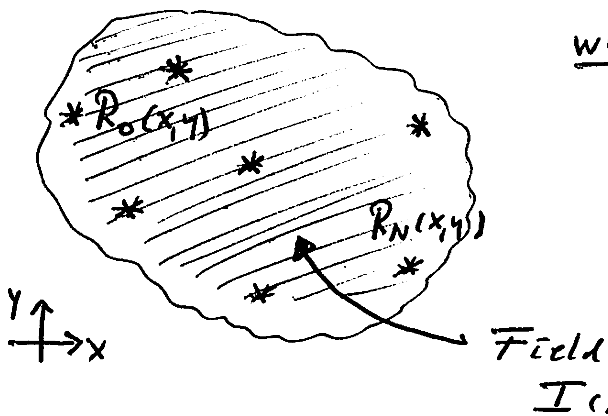
■ RECONSTRUCTION – Cont'd.

5) Using a Radial Basis Function Method

→ Intensity Field defined as:

$$I(x,y) = \sum_{i=1}^{N} c_i R_i(x,y) ,$$

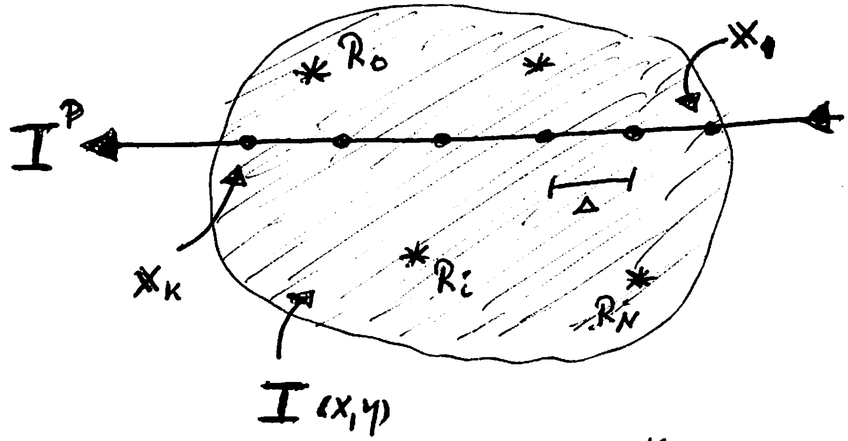


where: • $R_i(x,y)$ is the i^{th} radial basis function,

• c_i, $i=1,\dots,N$, are the N unknown coefficients.

$\ast$ center of a basis function

→ Setting up linear equation system for c_i's:



I^P = "projected intensity"

• sample locations along ray: $\ast_1 \dots \ast_K$

$$\Rightarrow I^P = \sum_{j=1}^{K} \Delta \cdot I(\ast_j) ,$$

('Radon')

$$\text{where } I(\ast_j) = \sum_{i=1}^{N} c_i R_i(\ast_j) ,$$

$$\Rightarrow I^P = \Delta \cdot \sum_{j=1}^{K} \sum_{i=1}^{N} c_i R_i(\ast_j) .$$

■ RECONSTRUCTION - Cont'd.

6) Types / Properties of Radial Basis Functions

i) Global vs. Localized basis functions:

→ Localized ⇒ Sparse linear system (BUT: least-squares system!)

ii) Example of global basis function:

→ Hardy's basis function:

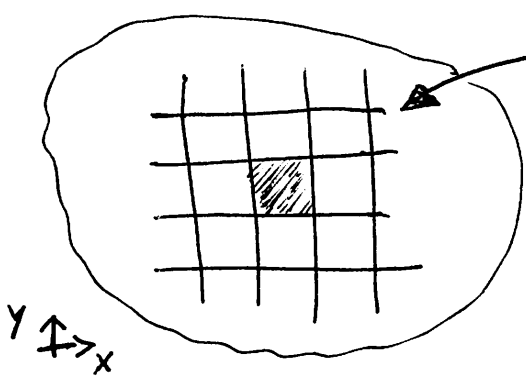
$$R_i(x,y) = \sqrt{c^2 + d_i^2(x,y)} \quad , \quad c^2 > 0 \, ,$$

where $d_i^2(x,y) = (x - x_i)^2 + (y - y_i)^2$,

(x_i, y_i) being the center of R_i .

iii) Example of simple(st) Local basis function:

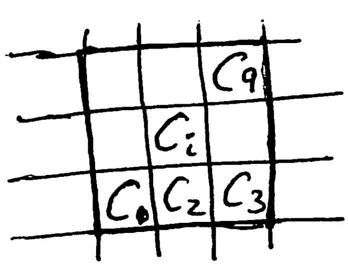
→ Set of BOX basis functions for Cartesian grid:



- cell $c_{i,j}$ used to discretize domain of field $I(x,y)$
- box function associated with cell $c_{i,j}$:

$$R_{i,j} = \begin{cases} 1 & \text{if } * \in c_{i,j} \\ 0 & \text{otherwise} \end{cases}$$

- Can also use single-index notation:

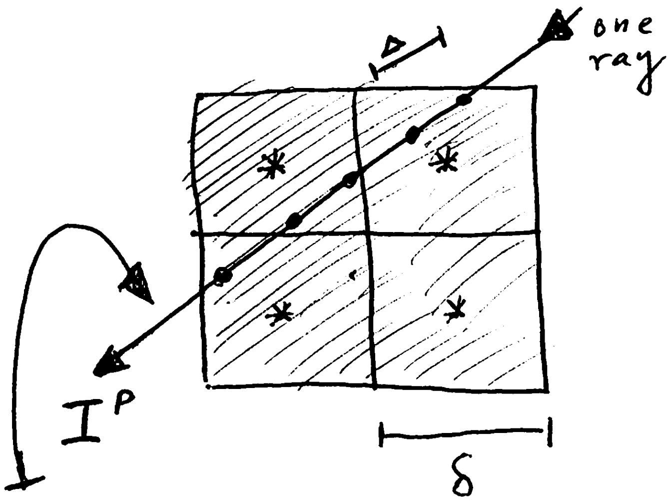


⇒ cell C_i has associated box function $R_i(*)$:

$$R_i(*) = \begin{cases} 1 & \text{if } * \in C_i \\ 0 & \text{otherwise} \end{cases}$$

RECONSTRUCTION – Cont'd.

7) The BOX function basis (OR: BI-/TRI-LINEAR BASIS...)



✳ centers of BOX functions

• sample locations along a specific ray

➜ spacings Δ and δ approx. the same...

Should use 3D version of Bresenham's line drawing algorithm to determine 3D cells containing '•'.

➜ Several samples '•' of many rays can lie in the grid cell C_i associated with BOX basis function $R_i(x,y)$.

➜ The intensity field function $I(x,y)$ is the piecewise-constant function implied by the Cartesian grid and its associated box functions:

$$I(x,y) = \sum_{i=1}^{N} c_i \, R_i(x,y) \, ,$$

where N = no. of grid cells,

c_i = coefficient of cell C_i = constant intensity value of cell C_i.

➜ DEPENDING ON RESOLUTION / SAMPLING VALUES, THE SYSTEM FOR THE UNKNOWN c_i-VALUES CAN BE OVER- OR UNDER-DETERMINED; USE LEAST-SQUARES SOLUTION APPROACH! BH

Bernd Hamann

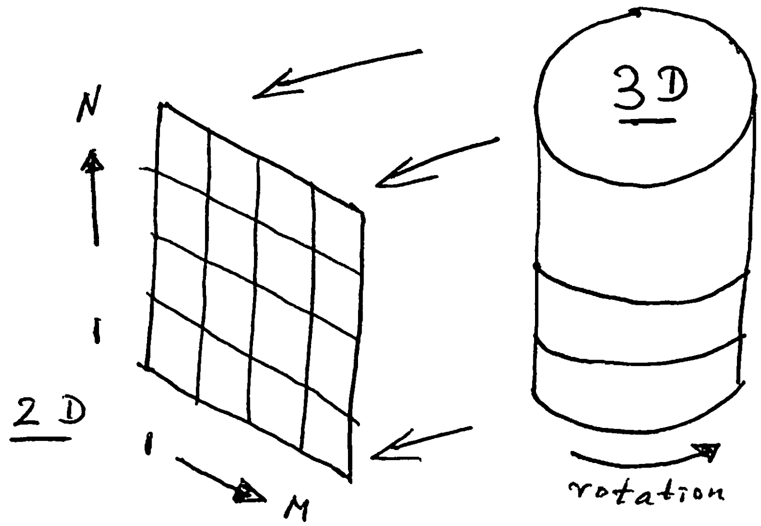
Stratovan

7-10-2017

■ RECONSTRUCTION – Cont'd.

8) Additional Issues / Thoughts

→ **Resolution / Efficiency:**



- **180** projections ($0°, 1°, 2°, \ldots, 179°$)
- Each 2D projection: resolution $M \times N$
- Typically required **3D** resolution of volume data: 512^3
- Ideally: Guarantee an **OVER-DETERMINED SYSTEM** ("for the 512^3 unknowns")

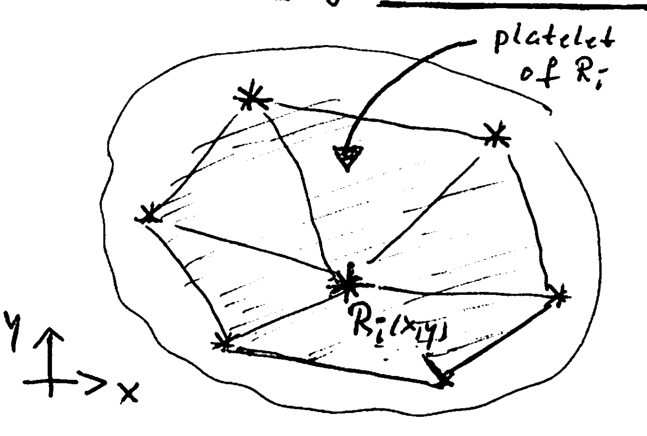
→ Iterative **Refinement**:

i) Compute a 3D reconstruction using **SMALL** M, N, no. of projections.

ii) Compute a better reconstruction using **LARGER** M, N, no. of projections.

iii) Stop refining when 3D reconstruction no longer improves.

→ **Triangulation-based** piecewise-linear basis functions:



- basis function $R_i(x,y)$ defined for shaded platelet
- $R_i(x,y)$ varies between 0 and 1 over platelet (being 1 at center vertex, being 0 at center's neighbors)
- Reconstruction $I(x,y) = \sum c_i R_i(x,y)$ to be evaluated on Cartesian grid.