Bernd Hamann

Stratovan

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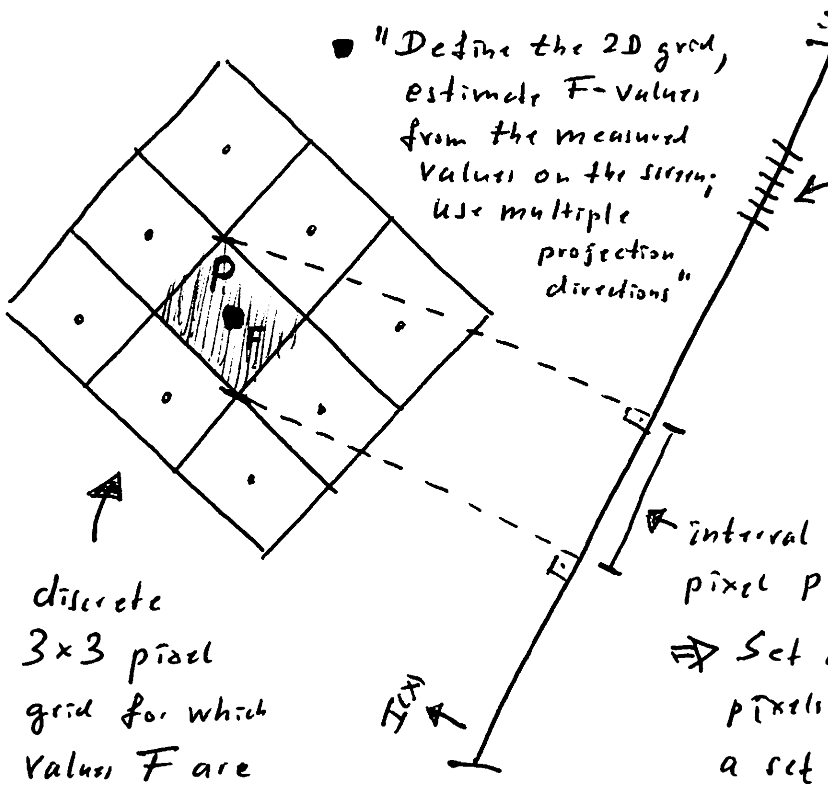
Simple Radon Transform, Reconstruction & "Splatting"

[See also: Roger Crawfis, "Image-Aligned Sheet-Based splatting" – IASB splatting ⇒ No BLEEDING, No NOISE]

→ Adopt a splatting view:

- "Define the 2D grid, estimate F-values from the measured values on the screen; use multiple projection directions"

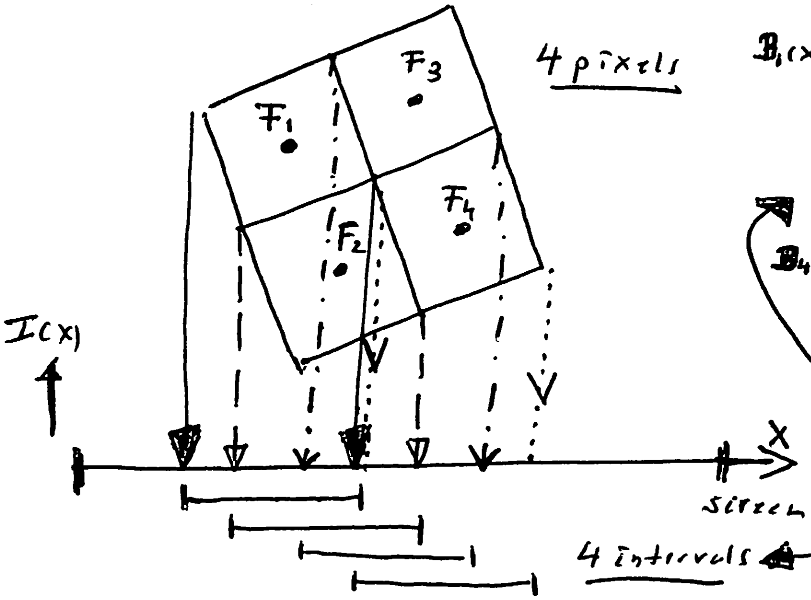


← screen itself is a discretized line – and accumulated intensity values define the values at each midpoint of those small line segments.

← interval to which pixel P projects ⇒ Set of all 3×3 pixels projects to a set of intervals (on the screen) that overlap.

discrete 3×3 pixel grid for which values F are to be reconstructed

→ Simpler 2×2 case:

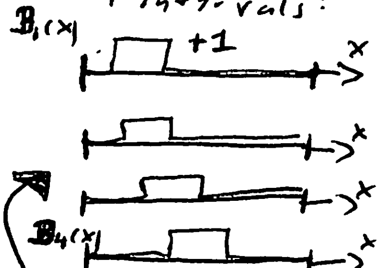


4 pixels

screen

4 intervals

1) Define Box functions for the 4 intervals:



2) Simple accumulation of function values F_i via Radon:

"Intensity function along x-axis" =

$$I(x) = \sum_{i=1}^{4} F_i \cdot B_i(x)$$

3) GIVEN: $I(x)$ ⇒ COMPUTE: F_i

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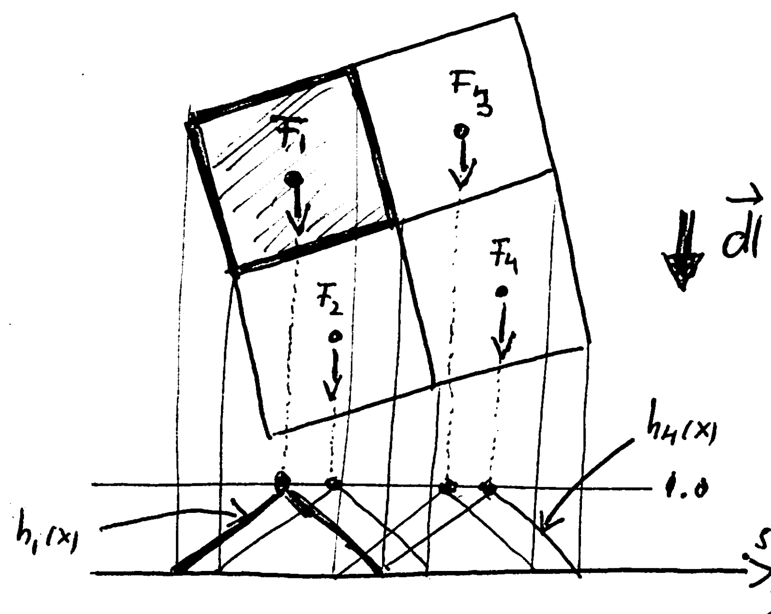
Reconstruction – Splatting View – Radon – Cont'd.

→ **OR:** Use piecewise linear hat functions:

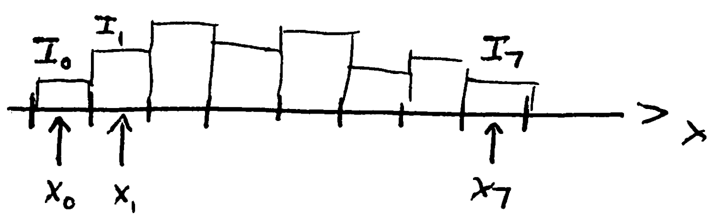


$$\Rightarrow I(x) = \sum_{i=1}^{4} F_i \cdot h_i(x)$$

given: $I(x)$, $h_i(x)$

computed: F_i

Actually, $I(x)$ is provided as a set of discrete values:



⇒ equations:

$$I_0 = \sum_{i=1}^{4} F_i \cdot h_i(x_0)$$

$$\vdots$$

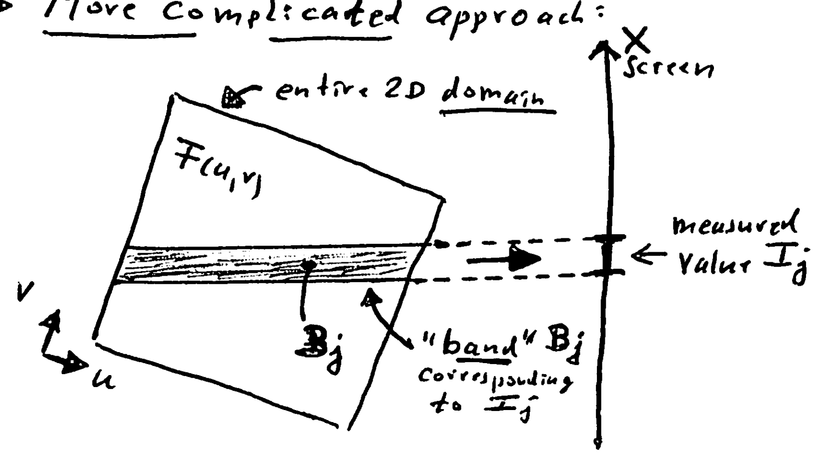
$$I_7 = \sum_{i=1}^{4} F_i \cdot h_i(x_7)$$

$$\begin{pmatrix} I_0 \\ \vdots \\ I_7 \end{pmatrix} = \begin{pmatrix} h_1(x_0) & h_2(x_0) & \dots & h_4(x_0) \\ & & \vdots & \\ h_1(x_7) & h_2(x_7) & \dots & h_4(x_7) \end{pmatrix} \begin{pmatrix} F_1 \\ \vdots \\ F_4 \end{pmatrix}$$

- **Plus:** multiple directions $\vec{d}$!

→ Solve with least-squares method.

→ **More complicated approach:**

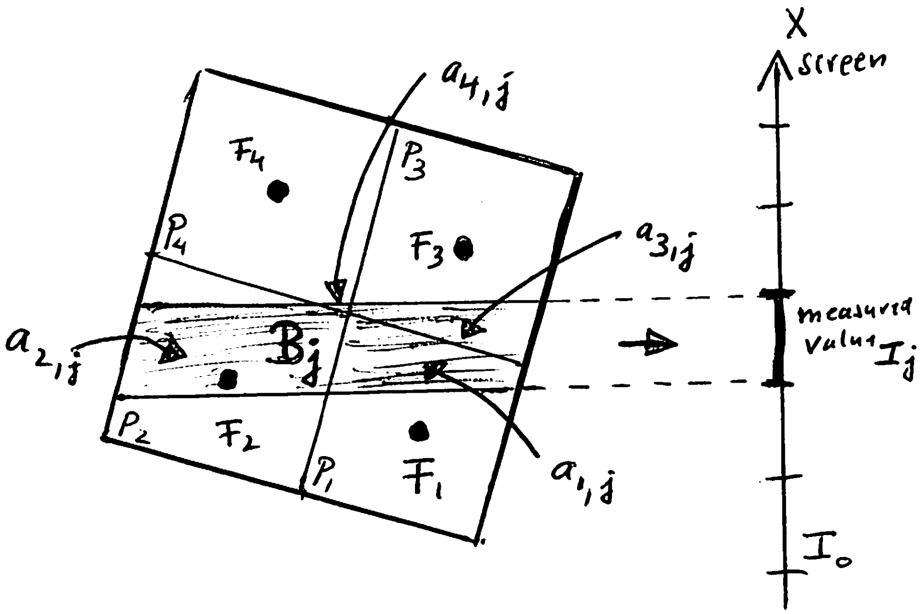


1) The unknown function to be reconstructed is $F(u,v)$ – understood as the function eventually represented by "2D grid."

2) $$I_j = \iint_{B_j} F(u,v) \cdot du\, dv$$

Reconstruction – Cont'd.

→ Actual reconstruction for 2D grid:



1) Consider the case of BOX functions associated with the center/domain of each 2D pixel; the function $F(u,v)$ is the piecewise-constant function

$$F(u,v) = \sum_{i=1}^{4} F_i \cdot B_i(u,v) \; .$$

2) The measured value I_j is defined as

$$I_j = \iint_{B_j} F(u,v) \cdot du\, dv$$

3) The pixels $P_1, \ldots, P_4$ have intersection regions with band B_j of sizes/areas $a_1, \ldots, a_4$, respectively.

4) Thus: $I_j = \sum_{i=1}^{4} a_{i,j} \cdot F_i$ or, in matrix form:

unknowns: the F_i-values

$$\begin{pmatrix} I_0 \\ \vdots \\ I_{N-1} \end{pmatrix} = \begin{pmatrix} a_{1,0} & \cdots & a_{4,0} \\ \vdots & & \vdots \\ a_{1,N-1} & \cdots & a_{4,N-1} \end{pmatrix} \begin{pmatrix} F_1 \\ \vdots \\ F_4 \end{pmatrix}$$

⇒ solve with LEAST SQUARES!

! 5) ISSUE: IT IS EXPENSIVE to compute the areas $a_{i,j}$!

6) One could also consider HAT FUNCTIONS instead of BOX functions; this would further increase computational cost.

!! 7) RESEARCH: WHAT QUALITY OF RECONSTRUCTION IS NECESSARY & SUFFICIENT FOR CLASSIFICATION?

!!! 8) BUT: WE CAN PRE-COMPUTE! ☺

BH