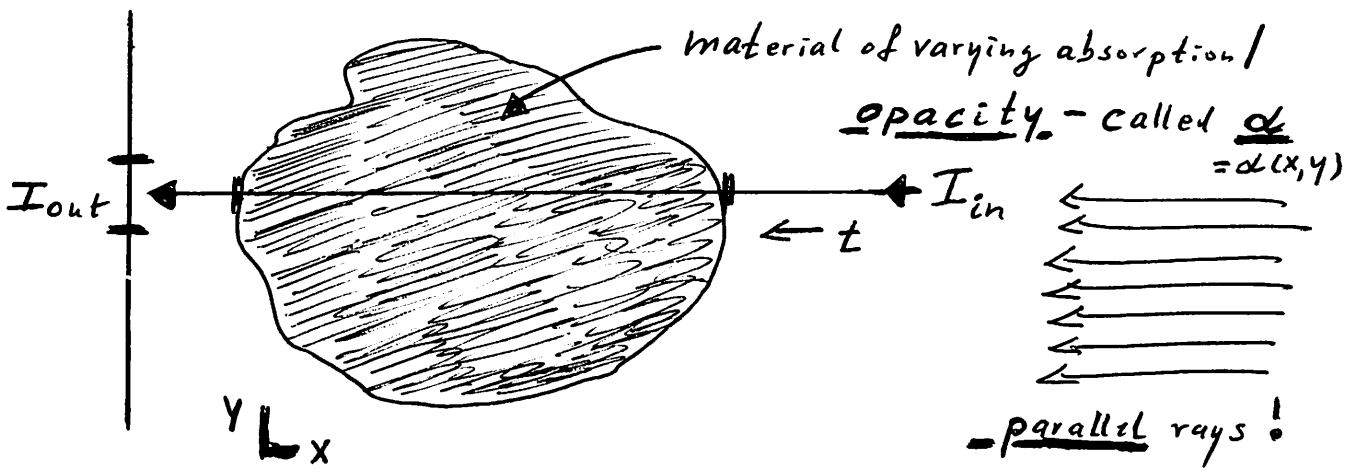
Bernd Hamann
Stratovan

10/16/2017

■ Towards High-quality Reconstruction: OPACITY & Absorption

→ Using **OPACITY** to model absorption of non-homogeneous materials!

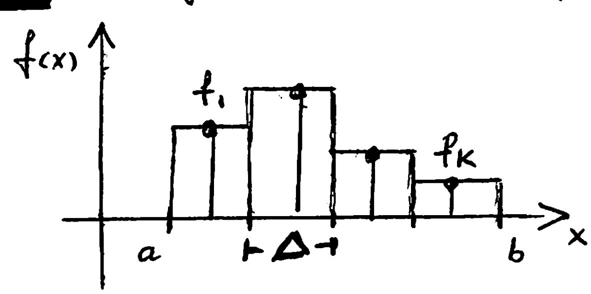
- Consider 2D slice example:



⇒ Solution of differential equation modeling light propagation and absorption:

$$(i) \qquad I_{out} = I_{in} \cdot e^{-\int_a^b \alpha(t)\, dt} + \int_a^b I(t) \cdot e^{-\int_t^b \alpha(\tilde{t})\, d\tilde{t}}\, dt$$

{ **NOTE**: Integration done via piecewise constant function:



$$\int_a^b f(x)\, dx = \Delta \sum_{i=1}^{K} f_i$$

}

⇒ Approximate solution for I_{out}:

$$(ii) \qquad I_{out} = I_{in} \cdot e^{-\Delta \sum_{i=1}^{K} \alpha_i} + \Delta \sum_{i=1}^{K} \left(I_i \cdot e^{-\Delta \sum_{j=i}^{K} \alpha_j} \right)$$

[redacted]

Here:
- K = # Bresenham pixels
- Δ = average distance between two consecutive Bresenham pixels

Bernd Hamann
Stratovan

▪ Reconstruction Cont'd. – Absorption ONLY

(i) $$I_{out} = I_{in} \cdot e^{-\int_a^b \alpha(t)\,dt}$$

(ii) $$I_{out} = I_{in} \cdot e^{-\Delta \sum_{i=1}^{K} \alpha_i}$$

$$\hookrightarrow \frac{I_{out}}{I_{in}} = e^{-\Delta \sum_{i=1}^{K} \alpha_i}$$

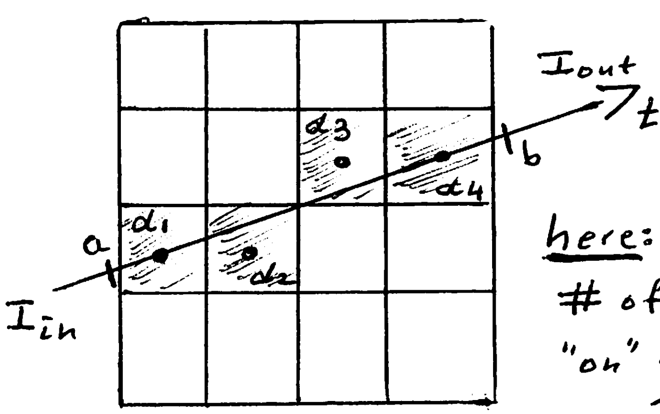
$$\hookrightarrow \ln\left(\frac{I_{out}}{I_{in}}\right) = -\Delta \sum_{i=1}^{K} \alpha_i$$

$$\hookrightarrow \ln(I_{out}) = \ln(I_{in}) - \Delta \sum_{i=1}^{K} \alpha_i$$

$$\hookrightarrow \sum_{i=1}^{K} \alpha_i = \frac{\ln(I_{in}) - \ln(I_{out})}{\Delta}$$

$\Rightarrow$ Must determine $\underline{\alpha_i}$-values!

• **Example:**



here:
# of pixels "on" between 1 and 4

$\Rightarrow$ Issue: Definition of Δ

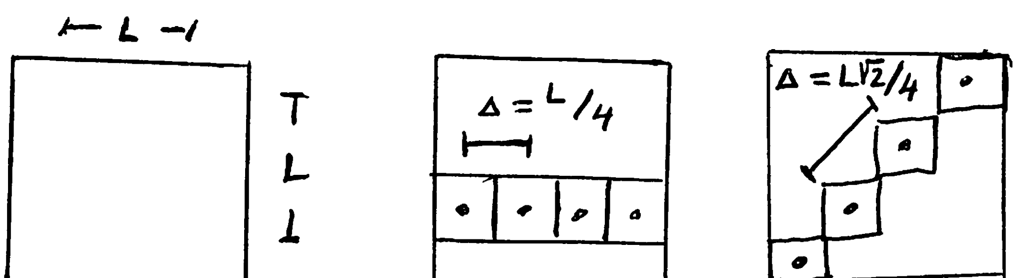
Possibility for example:

$$\Delta = (b-a)/\underline{4}$$

$$\Rightarrow \sum_{i=1}^{4} \alpha_i = \frac{\ln(I_{in}) - \ln(I_{out})}{\Delta}$$

$\Rightarrow$ USE ONE CONSTANT Δ-VALUE (FOR A RAY), EVEN WHEN PIXEL CENTERS OF CONSECUTIVE PIXELS HAVE DIFFERENT DISTANCES!

• Must consider extremal cases:



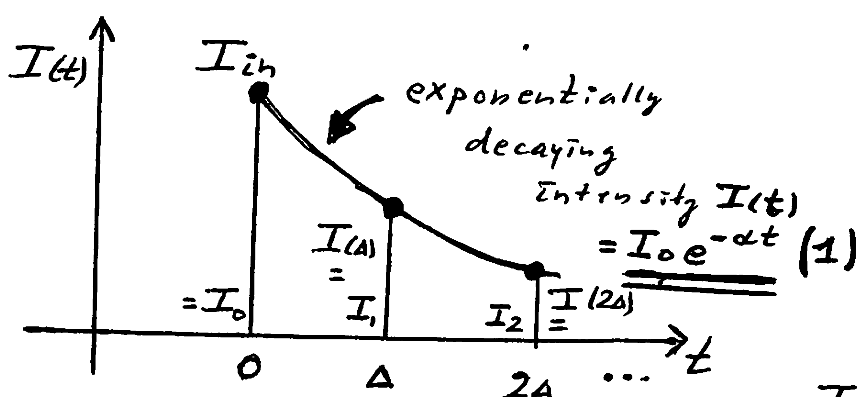
Bernd Hamann

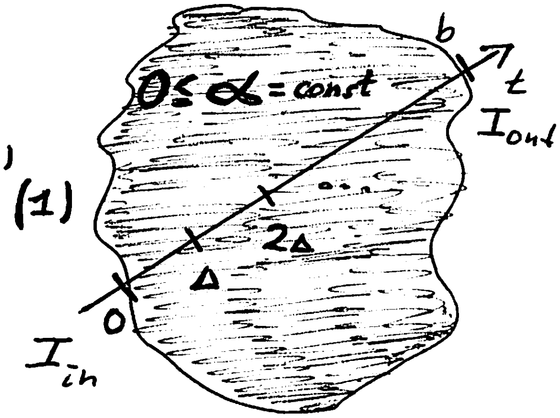
Stratovan

10/30/2017

Reconstruction Cont'd. – Absortion Model(s)

→ Considering a homogeneous material (with constant opacity α) and a "bundle of parallel light rays" passing through it, LIGHT INTENSITY DECREASES EXPONENTIALLY with distance traveled through the object:





$$\Rightarrow I_{in} = I_0$$

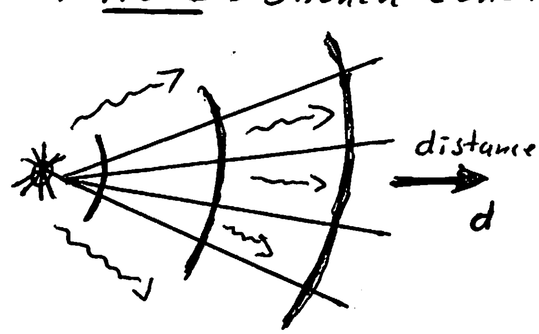
$$I_1 = I_0 e^{-\alpha\Delta}$$

$$\Rightarrow \frac{I_{j+1}}{I_j} = \frac{I_1}{I_0} = e^{-\alpha\Delta} = \frac{1}{e^{\alpha\Delta}}$$

$$\Rightarrow I_{j+1} = \frac{1}{e^{\alpha\Delta}} \cdot I_j \qquad (2)$$

[(2) is related to Sabella's back-to-front volume rendering formula!]

→ NOTE: Should consider effect of point light source:



i) 2D case: Photon / light density / intensity decreases LINEARLY with d.

ii) 3D case: ... decreases QUADRATICALLY with d.

⇒ MUST USE PROPER FACTOR BASED ON d WHEN USING (1) OR (2) FOR RECONSTRUCTION!

~ BH