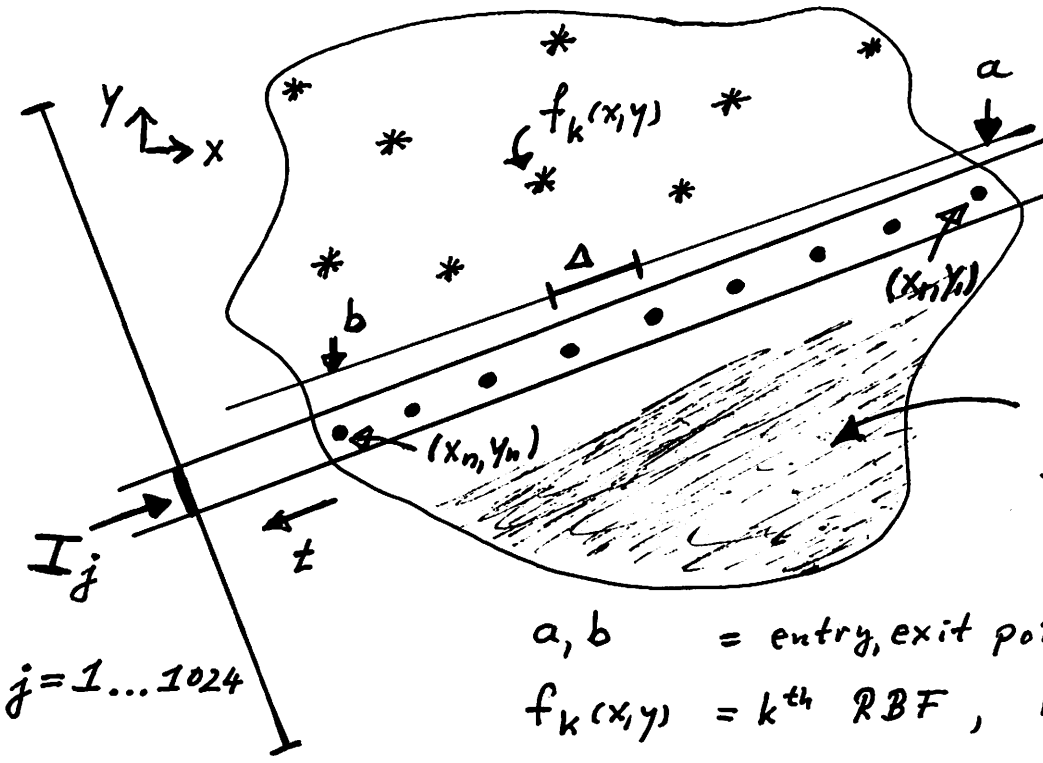
Reconstruction with Radial Basis Functions



* center of radial basis fct. "RBF"

reconstructed field function $\alpha = \alpha(x,y)$

a, b = entry, exit points

$f_k(x,y)$ = k^{th} RBF, $k = 1 \ldots K$

Δ = spacing along ray (constant)

I_o = incoming intensity

Goal: $$\boxed{\alpha(x,y) = \sum_{k=1}^{K} c_k f_k(x,y)}$$ → unknown: c_k

Use: $$I_j = I_o e^{-\int_a^b \alpha(t)\,dt}$$

$$\Rightarrow \underbrace{-\ln\left(I_j / I_o\right)}_{=: C_j} = \int_a^b \alpha(t)\,dt = \Delta \sum_{i=1}^{n} \alpha(x_i, y_i)$$

$j = 1 \ldots 1024$

Linear system: $$\boxed{\Delta \sum_{i=1}^{n} \left(\sum_{k=1}^{K} c_k f_k(x_i, y_i) \right) = C_j}$$

⇒ solve for unknown c_k's

BH

Stratovan

Reconstruction with RBFs - Cont'd.

Linear system precisely defined:

$\Rightarrow$ n_j samples "•" for ray j, $j = 1 \ldots 1024$

$$\Rightarrow \sum_{i=1}^{n_j} \left(\sum_{k=1}^{K} c_k f_k(x_{i,j}, y_{i,j}) \right) = \overline{C}_j, \quad j = 1 \ldots 1024,$$

where $\overline{C}_j = C_j / \Delta$

$$j=1 \quad \sum_{i=1}^{n_1} \left(\sum_{k=1}^{K} c_k f_k(x_{i,1}, y_{i,1}) \right) = \overline{C}_1$$

$$\ldots \qquad \ldots$$

$$j=1024 \quad \sum_{i=1}^{n_{1024}} \left(\sum_{k=1}^{K} c_k f_k(x_{i,1024}, y_{i,1024}) \right) = \overline{C}_{1024}$$

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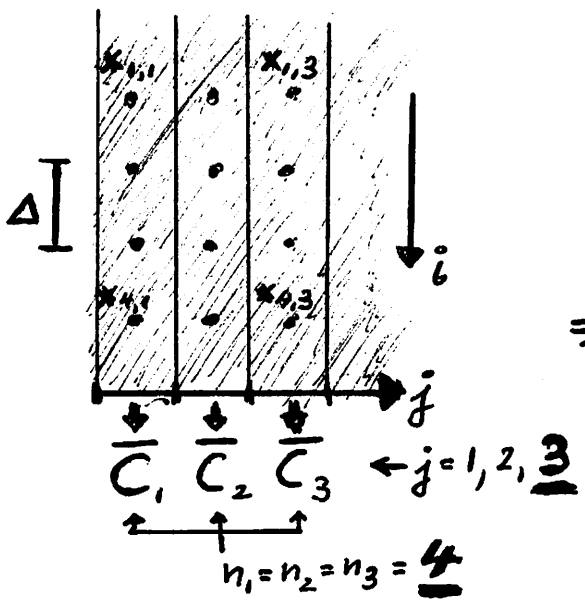
- Simple example:

$$\alpha(\mathbf{x}) = \alpha(x,y) = \sum_{k=1}^{2} c_k f_k(\mathbf{x}) \quad \{K=2\}$$



$$\overline{C}_1 = \sum_{i=1}^{4} \left(\sum_{k=1}^{2} c_k f_k(x_{i,1}) \right)$$

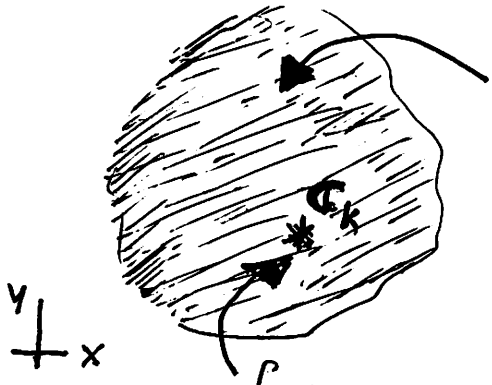
$$= \ldots = c_1 \sum_{i=1}^{4} f_1(x_{i,1}) + c_2 \sum_{i=1}^{4} f_2(x_{i,1})$$

$$\ldots$$

$$\Rightarrow \begin{bmatrix} \sum_{i=1}^{4} f_1(x_{i,1}) & \sum_{i=1}^{4} f_2(x_{i,1}) \\ \sum_{i=1}^{4} f_1(x_{i,2}) & \sum_{i=1}^{4} f_2(x_{i,2}) \\ \sum_{i=1}^{4} f_1(x_{i,3}) & \sum_{i=1}^{4} f_2(x_{i,3}) \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} \overline{C}_1 \\ \overline{C}_2 \\ \overline{C}_3 \end{bmatrix}$$

$\approx_{BH}$

■ RBF's – Cont'd.



Reconstruction:

$$\alpha = \alpha(x,y) = \sum_{k=1}^{K} c_k f_k(x,y)$$

- Issues:

! 1) K should be small.

2) Centers c_k should be placed optimally.

3) RBFs should have finite support.

→ Truncated Gaussian functions; B-spline basis functions (e.g., constant, linear, ..., cubic); "mixture" of varying-degree basis functions, including box/constant functions to model discontinuities.

4) "Shape" of RBF contours should be of a certain type to be able to model certain shapes in images:

etc.

→ modeling object boundaries

! 5) WHAT IS THE BEST BASIS FCT.?
(OR: What is the best set of basis fcts.?)

! 6) Progressively refine RBF representation by adding basis fcts.

7) Adapt wavelets/wavelet-like methods for hierarchical reconstruction.

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