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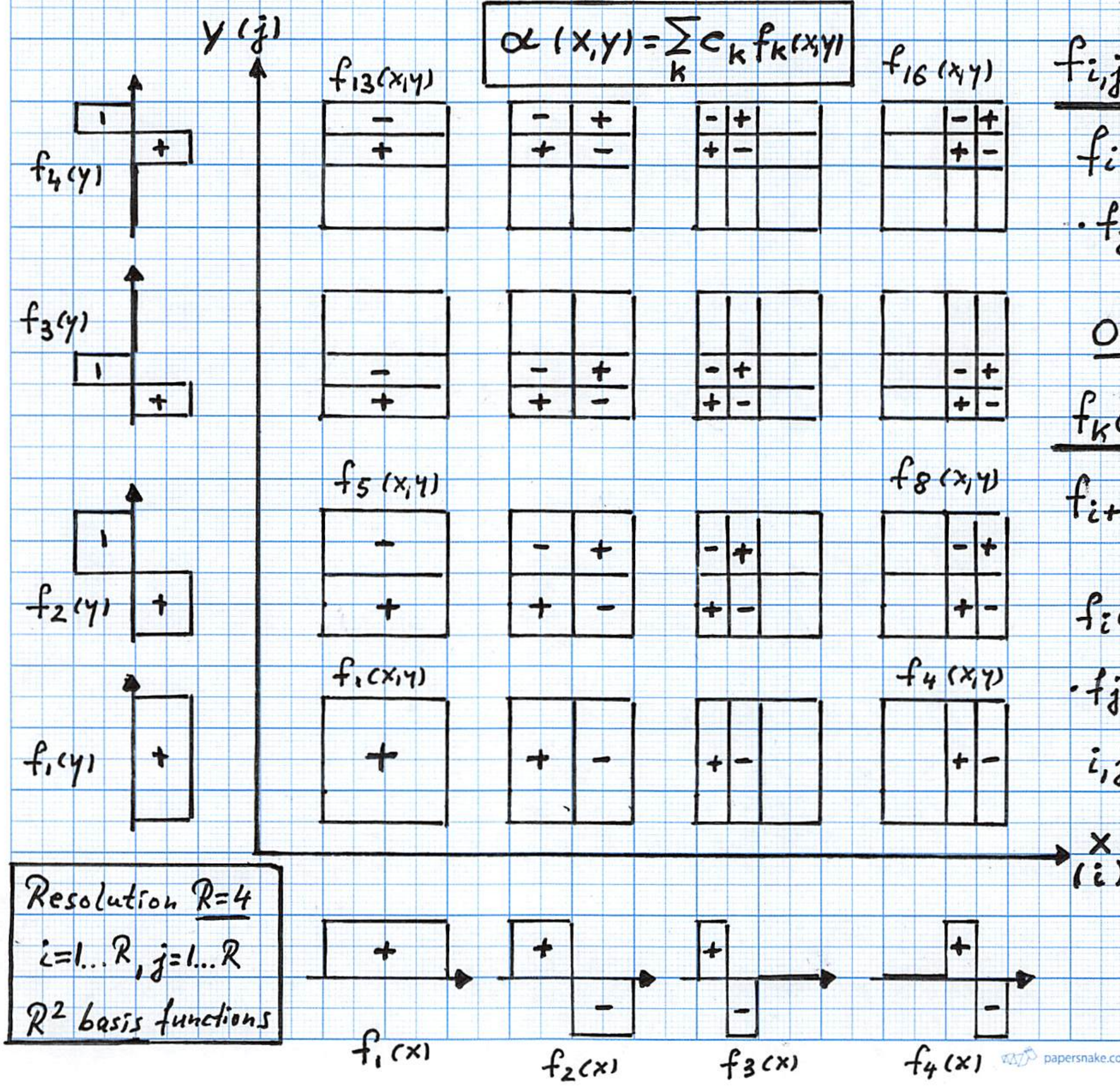
Multiresolution / Hierarchical Reconstruction Using Haar Wavelets

→ Use Haar wavelets to represent reconstruction $\alpha(x,y)$ and use them for an RBF-method!

- **Review:** Construction of bivariate tensor product Haar wavelet basis function:

$$\boxed{\alpha(x,y) = \sum_k c_k f_k(x,y)}$$



$f_{i,j}(x,y) = f_i(x) \cdot f_j(y)$

OR:

$f_k(x,y) = f_{i+(j-1)R}(x,y) = f_i(x) \cdot f_j(y)$

$i,j = 1 \ldots R$

Resolution $R=4$

$i = 1 \ldots R$, $j = 1 \ldots R$

R^2 basis functions

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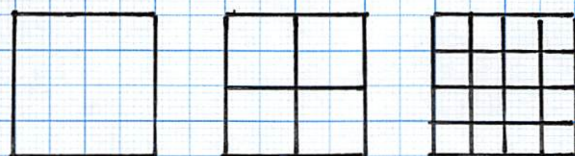
■ Reconstruction Using Haar Wavelets – Cont'd.

1) Once a Haar wavelet reconstruction is known, one can use it to represent an image in a progressive way with two basic structures:



... Power-of-2 Hierarchy

or:

... More general Wavelet Hierarchy

→ progressive refinement →

⇒ Must decide **which structure is** – overall – **"beneficial"** for segmentation, classification, ...

2) Once a Haar wavelet representation based on normalized basis functions (i.e., $\|f_{i,j}\|=1$) is known, one can compute a reconstruction using only those basis functions whose associated coefficients have absolute values $> \varepsilon$.

◄ 4-by-4 image in Haar wavelet representation $I(x,y) = \sum_{i,j=1}^{4} c_{i,j}\, f_{i,j}(x,y)$

{ Tensor product: $f_{i,j}(x,y) = f_i(x) \cdot f_j(y)$ }

⇒ $f_{i,j}(x,y)$ are the 16 normalized wavelet basis functions; only consider terms when $|c_{i,j}| > \varepsilon$. (⇒ Smoothing!)

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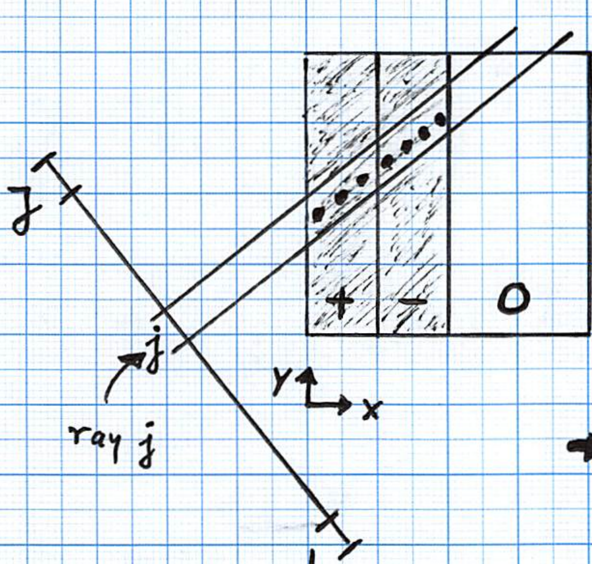
Reconstruction Using Haar Wavelets – Cont'd.

3) Using single-index notation for tensor product Haar wavelet basis functions, the bivariate reconstructed image is written as (opacity/absorption function): $\alpha(\mathbf{x}) = \sum_k c_k f_k(\mathbf{x})$.

➜ To define the matrix elements needed for solving the linear system for the unknown c_k-values, ONE MUST DETERMINE THE INTERSECTION BETWEEN A RAY AND THE NON-ZERO-VALUE REGION OF THE DOMAINS OF THE BASIS FUNCTIONS $f_k(\mathbf{x})$.

• Example: Basis function $f_3(x) \cdot f_1(y) = f_3(x,y)$ from page -1-



• = sample points "on ray j" in the non-zero-value region of $f_3(\mathbf{x})$ (= shaded region)

➜ One must (efficiently) determine when a sample point '•' is inside the non-zero-value region!

➜ The function value of a function f_k in the shaded region is defined by the normalization condition for f_k.

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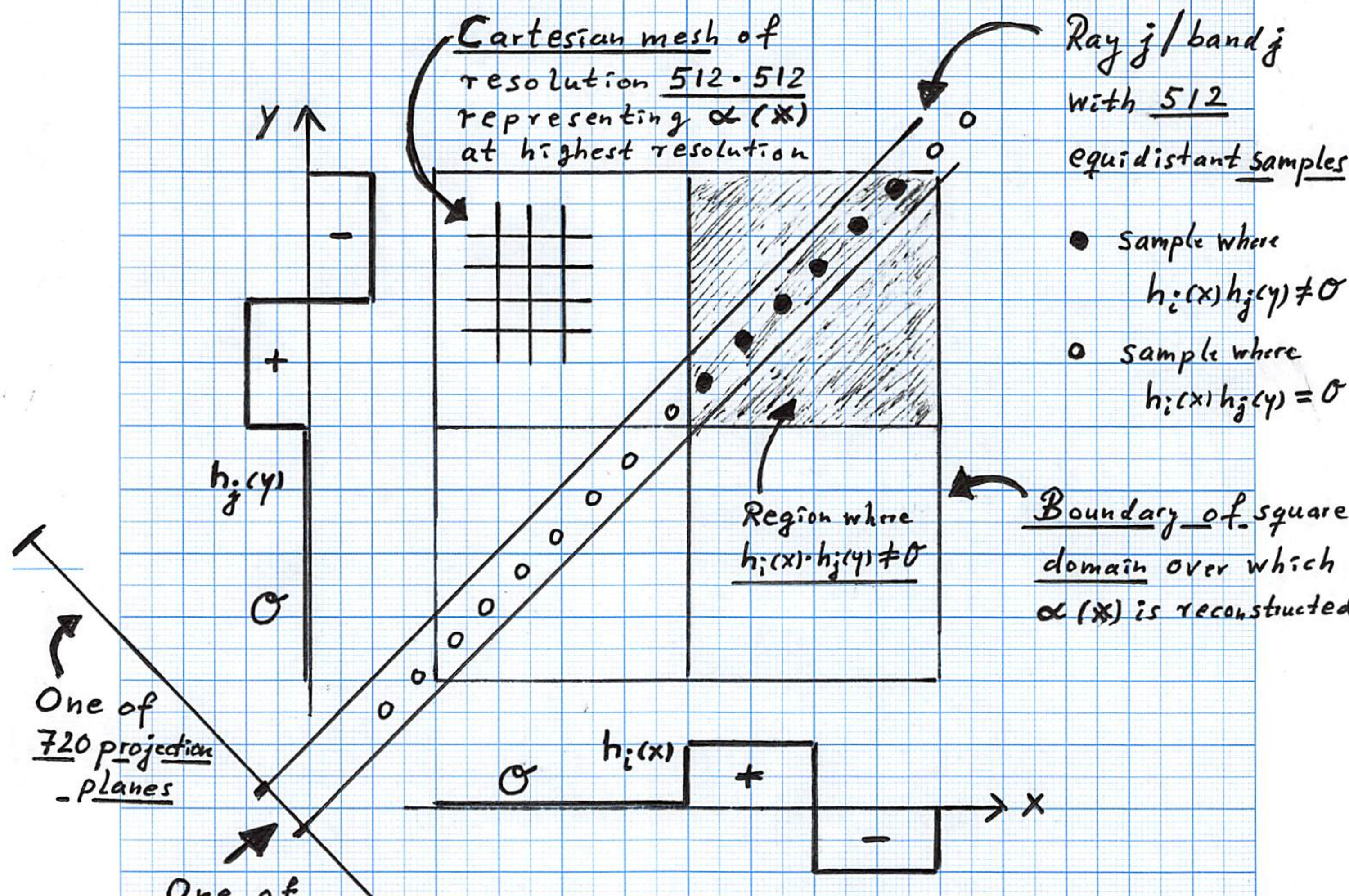
■ Reconstruction: Using Haar Wavelets as RBFs



→ **Here:** Specific tensor product basis function $f_k(\mathbf{x}) = h_i(x)\cdot h_j(y)$,

used in $\alpha(\mathbf{x}) = \sum_{k=1}^{512\cdot 512} c_k \cdot f_k(\mathbf{x})$

⇓

Total no. of rays/bands: 720·1024

⇒ System:

$$\underbrace{\left.\begin{bmatrix} \\ M \\ \\ \end{bmatrix}\right.}_{512\cdot 512 \text{ columns}} \cdot \underbrace{\begin{bmatrix} \\ \\ \\ \end{bmatrix}}_{512\cdot 512} = \begin{bmatrix} \\ \\ \\ \end{bmatrix} \quad 720\cdot 1024 \text{ rows}$$

⇒ Overdetermined system

Compute matrix value $m_{j,k}$ of $M = (m_{j,k})$:

for all 720·1024 rays/bands **do**

  for all 512·512 functions $f_k(\mathbf{x})$ **do**

   → determine samples '●'

   → compute $\sum_{\text{all '●'}} f_k(\bullet) =: m_{j,k}$

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