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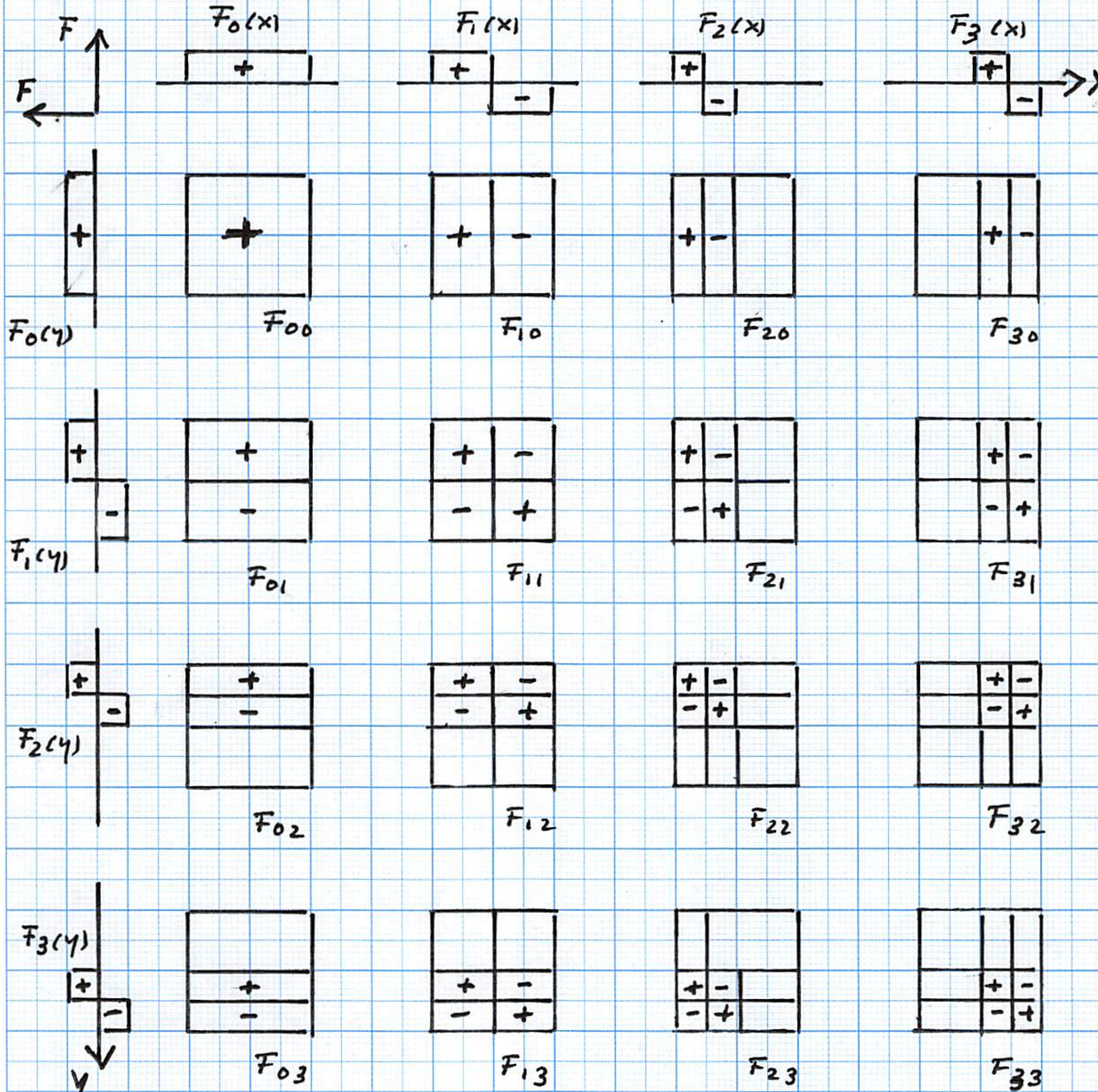
Using Haar Wavelets for Reconstruction and Local Reconstruction

- **Example:** Tensor Product Haar Wavelets for 4^2 Reconstruction



$$F_{ij} = F_{i,j}(x,y) = F_i(x) \cdot F_j(y)$$

$\Rightarrow$ Image (4-by-4) $I = \sum_{j=0}^{3} \sum_{i=0}^{3} c_{i,j} \cdot F_{i,j}(x,y)$

$\Rightarrow$ $c_{i,j}$-values approximated via sinogram!

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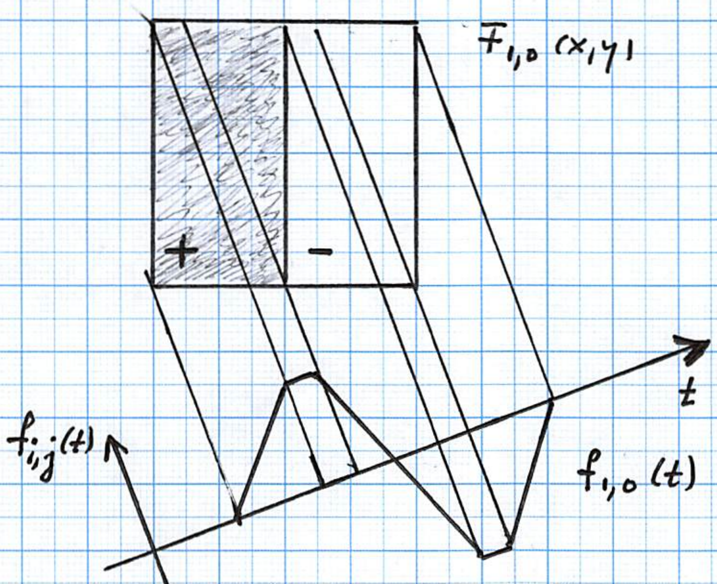
■ Haar Wavelets and (Local) Reconstruction - Cont'd.

⇒ Projections of all tensor product Haar wavelet basis functions are needed to approximate a sinogram $s(t)$ as a combination of projected Haar wavelet basis functions!

⇒ Haar wavelet basis functions project to functions $f_{i,j}(t)$ that have constant and linear segments.

→ **Example:** Projection of

$F_{1,0}(x,y) \rightarrow f_{1,0}(t)$



($F_{i,j}$ are NORMALIZED functions.)

- Approximate a given sinogram $s(t)$ as

$$s(t) = \sum_j \sum_i c_{ij} \cdot f_{ij}(t) .$$

- Compute the BEST APPROXIMATION of $s(t)$ via the normal equations. (See box basis function approach.)

- Use the computed c_{ij}-values to define image $I(x,y)$.

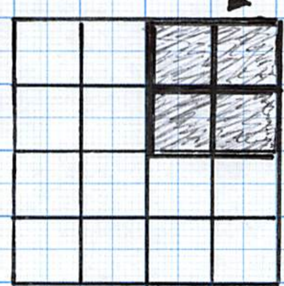
● • **NOTE:** Consider using only "the largest c_{ij}-values" — $(|c_{ij}| > \varepsilon)$ — to COMPRESS and REMOVE NOISE !!!

Haar Wavelets and (Local) Reconstruction – Cont'd.

→ Use of wavelet representation for **LOCAL** reconstruction:

- **Example** (4-by-4)

Local region of interest (2-by-2) – **ROI**



- Entire 4-by-4 image reconstructed using 16 coefficients:

$$I(x,y) = \sum_{j=0}^{3} \sum_{i=0}^{3} c_{i,j} \cdot F_{i,j}(x,y)$$

- Determine basis functions $F_{i,j}(x,y)$ that are NON-ZERO, by definition, in the ROI:

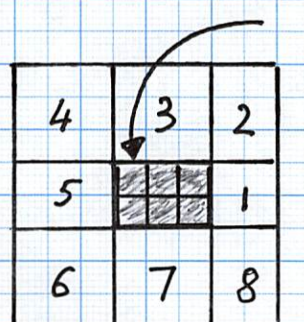
$F_{00}, F_{10}, F_{30}, F_{01}, F_{11}, F_{31}, F_{02}, F_{12}, F_{32}$

(⇒ These 9 functions define ROI completely.)

(• These "ROI-associated functions" have coefficients $c_{i,j}$; consider using only those coefficients – and basis functions – for which $|c_{i,j}| > \epsilon$)

(• **NOTE**: The following approach generally does **NOT** work!

Local ROI of resolution 3-by-2 (smallest-size pixels)

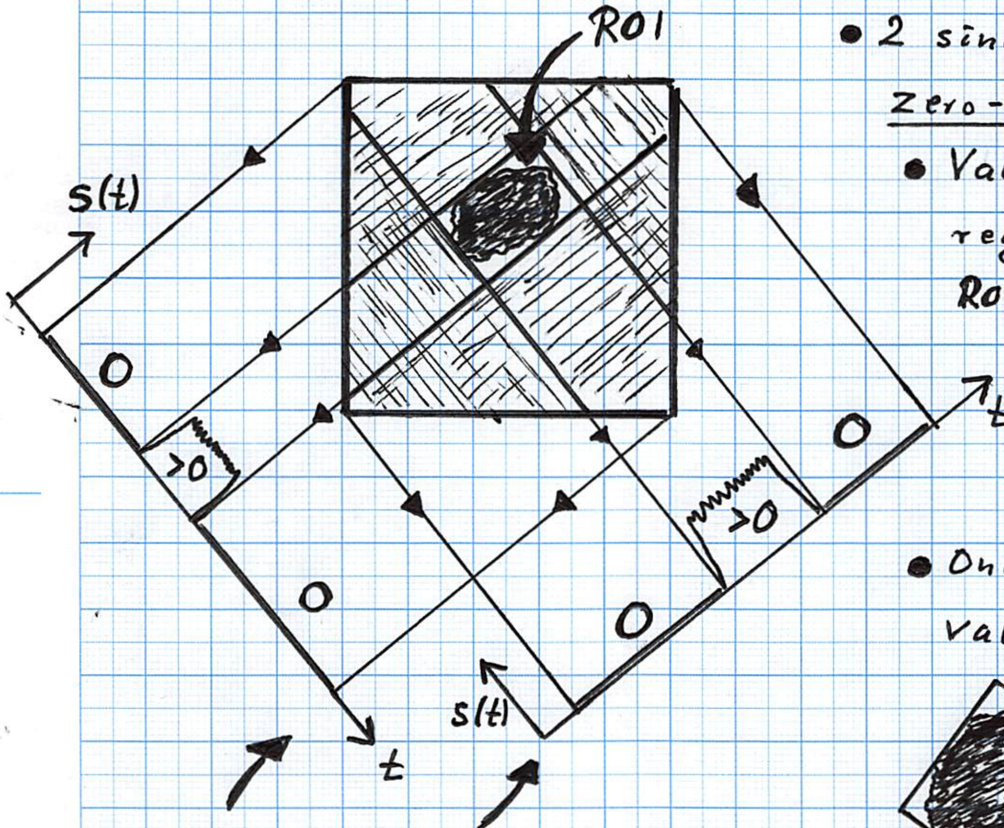


- ROI surrounded by 8 "quads"
- Box basis function for the image are now "Quad basis function" – being 1 inside the quad and 0 outside the quad.

• ISSUE: MUST **NOT** AVERAGE IMAGE VALUES BY JUST ONE VALUE PER SURROUNDING QUAD. !)

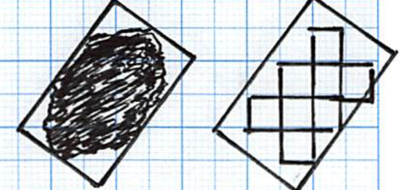
■ On Localized Reconstruction – When It Is Possible

→ Extreme case: ROI surrounded by homogeneous zero-density material(s)



Two sinograms defining ROI via computation of visual hull

- 2 sinograms with zero-value regions 'O'
  - Values 'O' define regions next to ROI where reconstruction is O.
  - ROI bounded by 'visual hull'
  - Only reconstruct values for ROI:



→ Visual hull of ROI and pixels inside

⇒ 1) Compute best approximations of sinograms only in '>0' regions.

2) Compute reconstructed pixel values only for pixels defining the ROI.

(Note: This approach can be generalized to the case when the ROI is surrounded by a constant-density material.) ~BH