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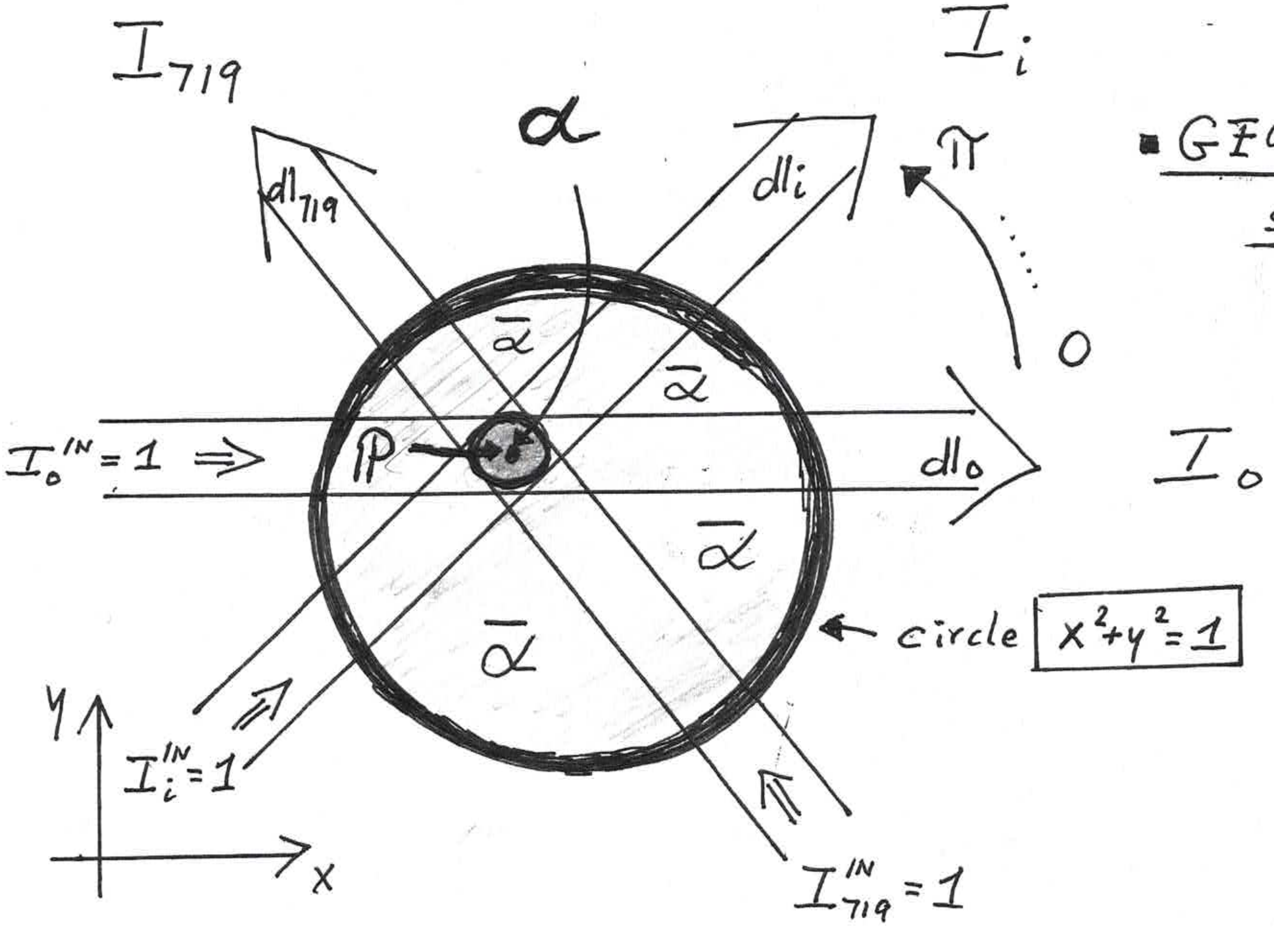
RECONSTRUCTION: DEFINING FUNCTION $\alpha(x,y)$
(DIRECTLY COMPUTING α FOR ANY POINT)



- GEOMETRY SET-UP

→ 720 angles of projection / ray directions

- angles: β_i, $\boxed{\beta_i = \frac{i}{720} \cdot \pi}$; $i = 0 \ldots 719$

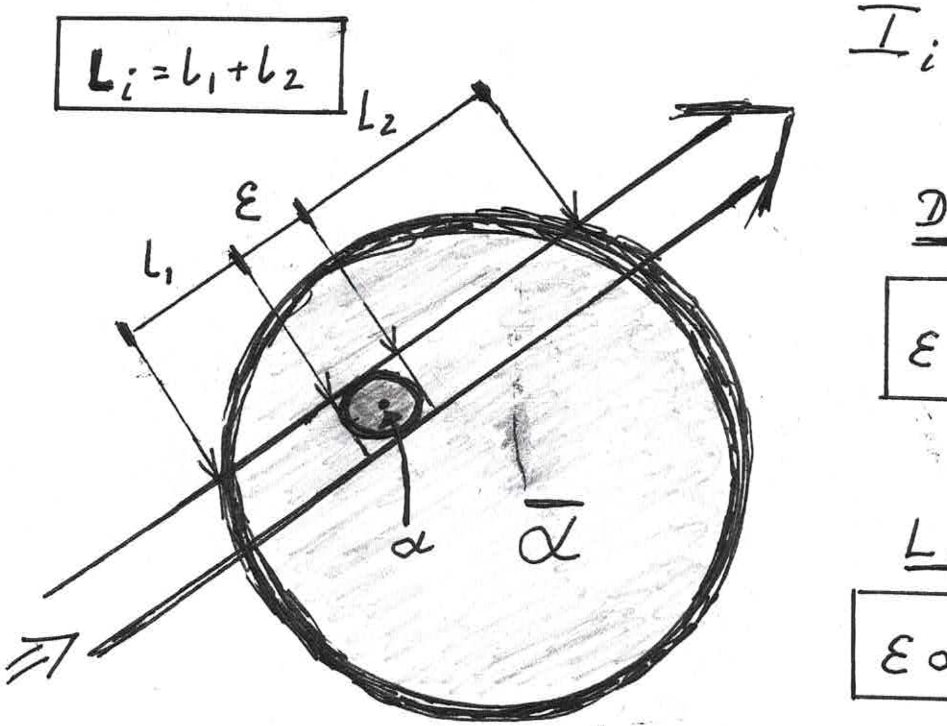
- directions: dl_i, $\boxed{dl_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix} = \begin{pmatrix} \cos(\beta_i) \\ \sin(\beta_i) \end{pmatrix}}$; $i = 0 \ldots 719$

→ 720 "sinogram measurements" I_i^{OUT} for point / $\boxed{\text{pixel } \mathbb{P} = \begin{pmatrix} x \\ y \end{pmatrix}}$; DEFINE: $\boxed{I_i = -\ln(I_i^{OUT})}$

→ 2 density / opacity values: $\alpha = \alpha(\mathbb{P})$ DESIRED

$\bar{\alpha}$ = "density in area surrounding $\mathbb{P}$"

LINEAR EQUATION SYSTEM



DEFINE:

$$\boxed{\varepsilon = \frac{1}{512}}$$

LAW:

$$\boxed{\varepsilon\alpha + L_i\bar{\alpha} = I_i}\ ; \quad i = 0 \ldots 719$$

$$\Rightarrow \left.\begin{array}{c} \varepsilon\alpha + L_0\bar{\alpha} = I_0 \\ \vdots \\ \varepsilon\alpha + L_{719}\bar{\alpha} = I_{719} \end{array}\right\} \Rightarrow \text{COMPUTE } \alpha \text{ VIA LEAST SQUARES}$$

$\Downarrow$

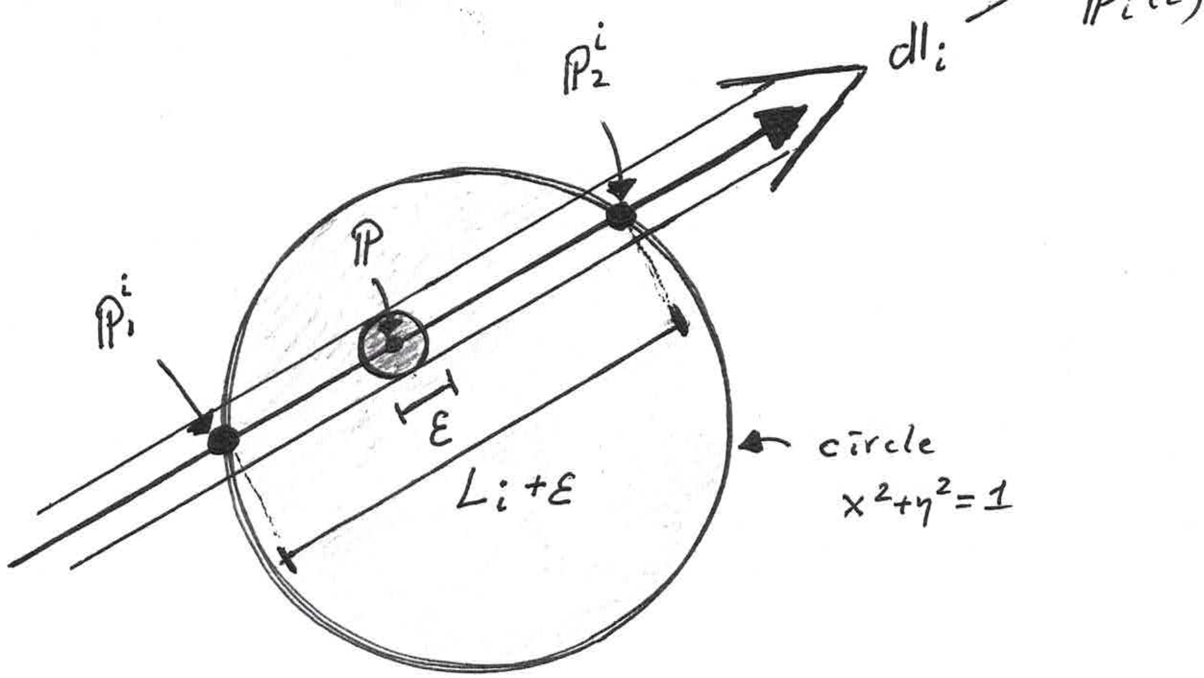
Matrix notation: $\begin{pmatrix} \varepsilon & L_0 \\ & \vdots \\ \varepsilon & L_{719} \end{pmatrix} \begin{pmatrix} \alpha \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} I_0 \\ \vdots \\ I_{719} \end{pmatrix}$

Solve $\begin{pmatrix} \varepsilon & \cdots & \varepsilon \\ L_0 & \cdots & L_{719} \end{pmatrix} \begin{pmatrix} \varepsilon & L_0 \\ & \vdots \\ \varepsilon & L_{719} \end{pmatrix} \begin{pmatrix} \alpha \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} \varepsilon & \cdots & \varepsilon \\ L_0 & \cdots & L_{719} \end{pmatrix} \begin{pmatrix} I_0 \\ \vdots \\ I_{719} \end{pmatrix}$

! RESULT: $\begin{pmatrix} 720\,\varepsilon^2 & \varepsilon \sum_0^{719} L_i \\ \varepsilon \sum_0^{719} L_i & \sum_0^{719} L_i^2 \end{pmatrix} \begin{pmatrix} \alpha \\ \bar{\alpha} \end{pmatrix} = \begin{pmatrix} \varepsilon \sum_0^{719} I_i \\ \sum_0^{719} L_i I_i \end{pmatrix}$!!!

2-by-2 system

■ INTERSECTION
OF RAY THROUGH $\mathbb{P}$
WITH DOMAIN BOUNDARY



DEFINE: $\boxed{L_i = \| \mathbb{P}_2^i - \mathbb{P}_1^i \| - \varepsilon}$

$\Rightarrow$ Compute intersections $\mathbb{P}_1^i$ and $\mathbb{P}_2^i$
between ray $\mathbb{P}_i(t) = \mathbb{P} + t \cdot \mathbb{d}_i = \begin{pmatrix} x + t u_i \\ y + t v_i \end{pmatrix}$
and circle $x^2 + y^2 = 1$:

$$(x + t u_i)^2 + (y + t v_i)^2 = 1$$

$$x^2 + 2t x u_i + t^2 u_i^2 + y^2 + 2t y v_i + t^2 v_i^2 = 1$$

$$t^2 (u_i^2 + v_i^2) + 2t (x u_i + y v_i) = 1 - x^2 - y^2$$

$$\{ u_i = \cos(\beta_i) \text{ and } v_i = \sin(\beta_i) \}$$

$$t^2 \quad + 2t (x \cos(\beta_i) + y \sin(\beta_i)) = 1 - x^2 - y^2$$

$$\ldots \Rightarrow t_1 = \ldots ; t_2 = \ldots \Rightarrow \mathbb{P}_1^i = \ldots ; \mathbb{P}_2^i = \ldots$$

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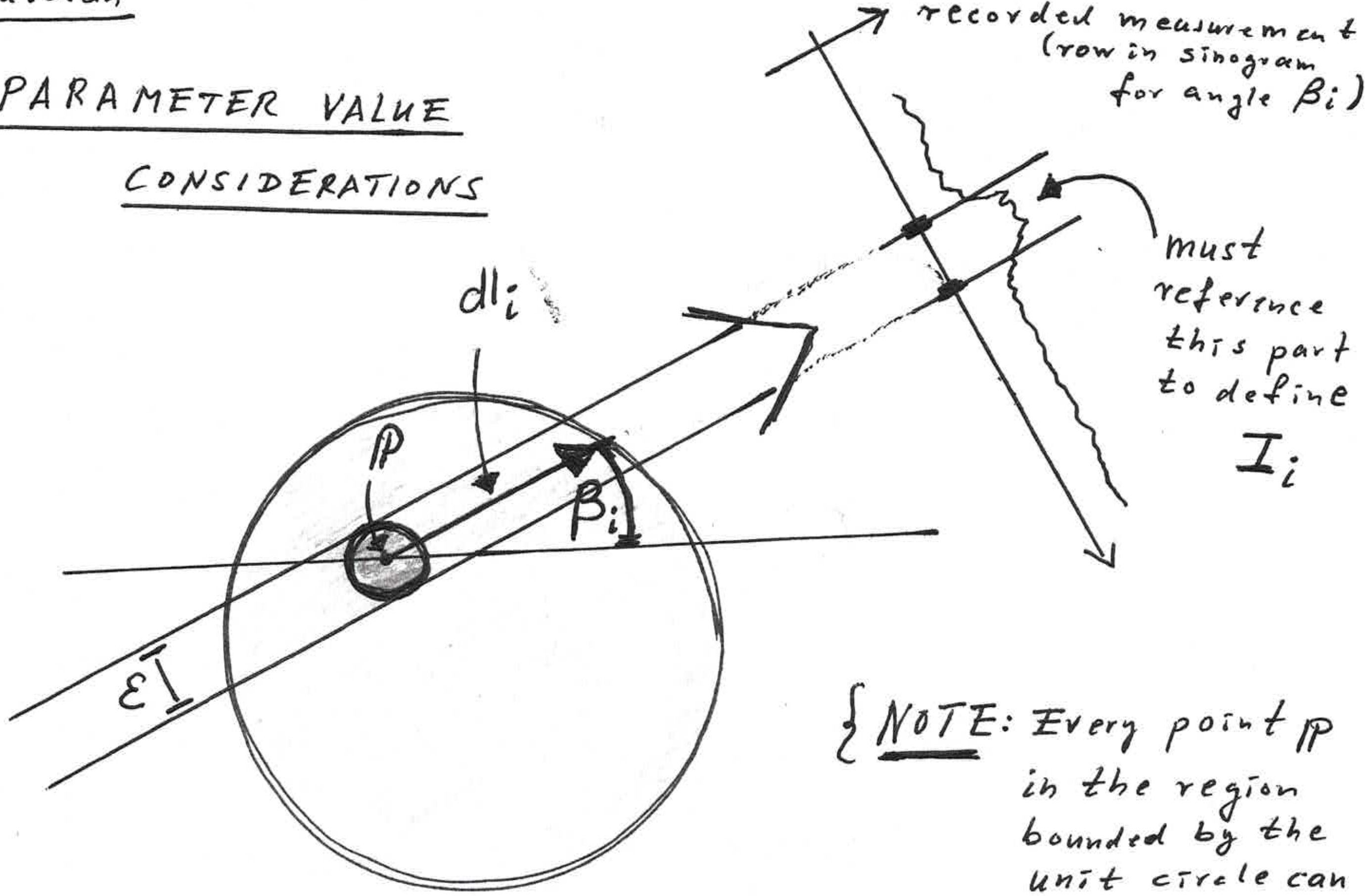
PARAMETER VALUE CONSIDERATIONS



{ **NOTE:** Every point $\mathbb{P}$ in the region bounded by the unit circle can be written as $\mathbb{P} = \binom{x}{y} = R \cdot \binom{\cos\beta}{\sin\beta}$, $0 \le R \le 1$ and $0 \le \beta \le 2\pi$. }

SPECIFICS:

- 720 angles of projection
- 1024 measurements per sinogram row
- 512 x 512 resolution of reconstruction (using points $\mathbb{P}$ on Cartesian mesh)
- $\varepsilon = \frac{1}{512}$ is "ok" (considering the other resolution values)

LIMIT CASE:

- $720 \Rightarrow \infty$
- $1024 \rightarrow \infty$
- $512 \times 512 \rightarrow \infty \times \infty$
- $\varepsilon \rightarrow 0$

$\Rightarrow$ towards analytical, continuous reconstruction ...

BH