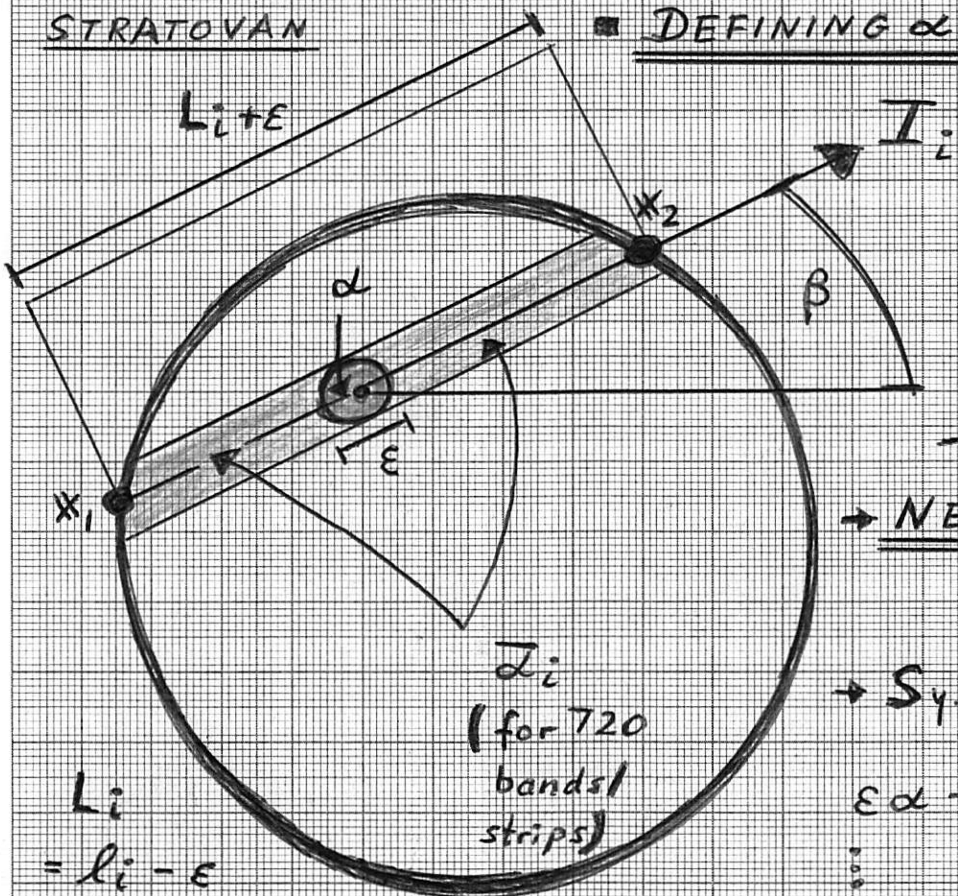
BERND HAMANN — 1 — 10/24/2018

STRATOVAN

■ DEFINING α FOR POINT (x,y)



$(= -\ln(I_i^{out}))$

→ NEXT PAGES: $L_i = \ldots \quad \varepsilon = \ldots$

→ $0^\circ \le \beta < 180^\circ$

→ **NEEDED: α**

$= \alpha(x,y)$

→ System:

$$\varepsilon\alpha + L_0\overline{\alpha}_0 = I_0$$
$$\vdots$$
$$\varepsilon\alpha + L_{719}\overline{\alpha}_{719} = I_{719}$$

L_i
$= \ell_i - \varepsilon$
$= \|x_2 - x_1\| - \varepsilon$

$$\Rightarrow \begin{bmatrix} \varepsilon & L_0 & & \\ \varepsilon & & L_1 & \\ \vdots & & & \ddots \\ \varepsilon & & & & L_{719} \end{bmatrix} \begin{bmatrix} \alpha \\ \overline{\alpha}_0 \\ \vdots \\ \overline{\alpha}_{719} \end{bmatrix} = \begin{bmatrix} I_0 \\ I_1 \\ \vdots \\ I_{719} \end{bmatrix}$$

$$M \qquad \vec{\alpha} \quad = \quad \vec{I}$$

→ underdetermined system!

(720 equations, 721 unknowns)

$$\Rightarrow \quad \underline{\vec{\alpha} = M^T(MM^T)^{-1}\vec{I}} \quad \text{(solution closest to } \vec{0}\text{)}$$

$$\underline{\text{or}} \quad \underline{\vec{\alpha} = \vec{p} + M^T(MM^T)^{-1}(\vec{I} - M\vec{p})} \quad \Rightarrow \underline{\alpha = \ldots}$$

(solution closest to $\vec{p}$)

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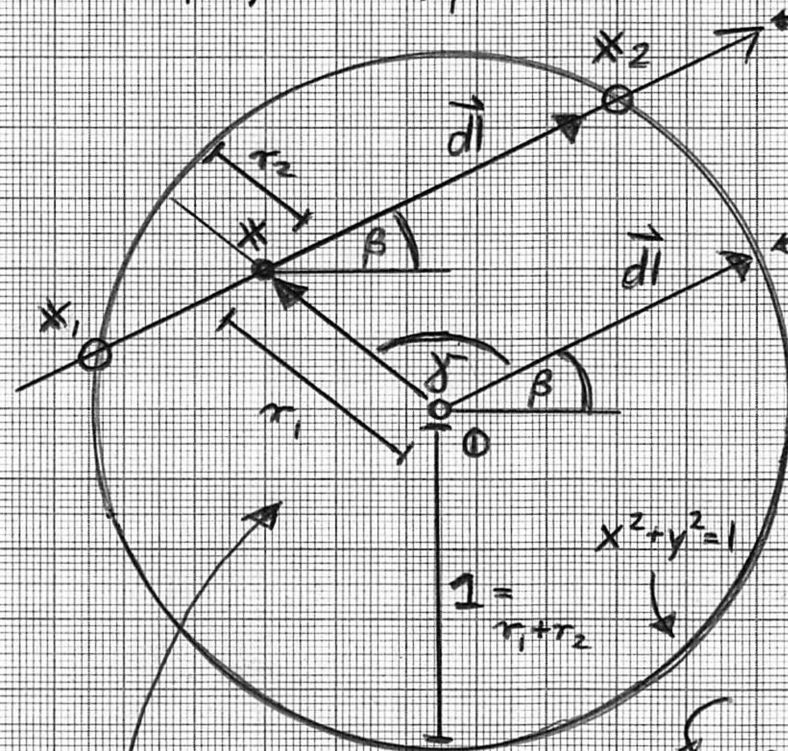
■ DEFINING α FOR POINT (x,y)

→ NEEDED: $\ell = \|\mathbf{x}_2 - \mathbf{x}_1\|$

$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \vec{dl} = \begin{pmatrix} d_x \\ d_y \end{pmatrix}$

$\mathbf{x}(t) = \mathbf{x} + t\,\vec{dl}$



$\vec{dl} = \begin{pmatrix} d_x \\ d_y \end{pmatrix} = \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}$

$= \begin{pmatrix} c\beta \\ s\beta \end{pmatrix}$

Intersections $\mathbf{x}_1$ and $\mathbf{x}_2$ satisfy:

$$(x + t c\beta)^2 + (y + t s\beta)^2 = 1$$

$$t^2 (c^2\beta + s^2\beta) + t\,2\,(x c\beta + y s\beta) = 1 - (x^2+y^2)$$

$$t^2 + t\,2\,[x, y]\begin{bmatrix} d_x \\ d_y \end{bmatrix} = r_2$$

$$t^2 + t\,2\,\mathbf{x}\cdot dl + (\mathbf{x}\cdot dl)^2 = r_2 + (\mathbf{x}\cdot dl)^2$$

$$(t + (\mathbf{x}\cdot dl))^2 = r_2 + (\mathbf{x}\cdot dl)^2$$

$$t_{1,2} = -\mathbf{x}\cdot dl \;+/-\; \sqrt{r_2 + (\mathbf{x}\cdot dl)^2}$$

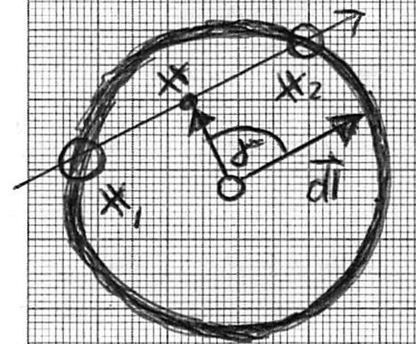
$\cos\gamma = c\gamma$

$= \dfrac{\mathbf{x}\cdot dl}{\|\mathbf{x}\|\,\|dl\|}$

$= (\mathbf{x}\cdot dl)/r_1$

$\mathbf{x}\cdot dl = r_1 c\gamma$

$$\underline{\underline{t_{1,2} = -r_1 c\gamma \;+/-\; \sqrt{r_2 + (r_1 c\gamma)^2}}}$$



$\Longleftrightarrow$

$$\Rightarrow \underline{\underline{\mathbf{x}_{1,2} = \mathbf{x} + t_{1,2}\,\vec{dl}}}$$

$$\Rightarrow \underline{\underline{\|\mathbf{x}_2 - \mathbf{x}_1\| = \sqrt{(t_2 - t_1)^2} = 2\sqrt{r_2 + (r_1 c\gamma)^2} = \ell}}$$

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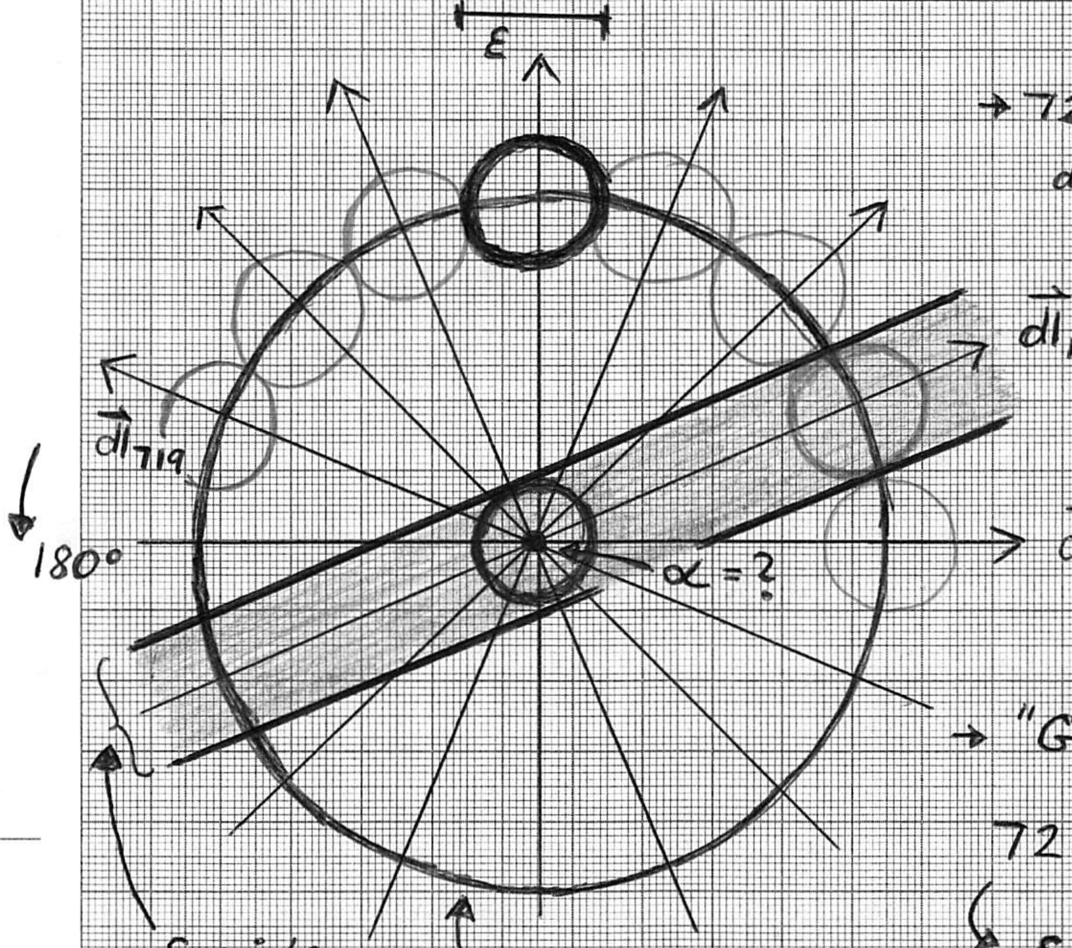
▪ DEFINING α FOR POINT (x,y)



→ 720 projection directions $\vec{dl}_i$

→ "GOOD" ε-value:

$720\,\varepsilon = \pi$

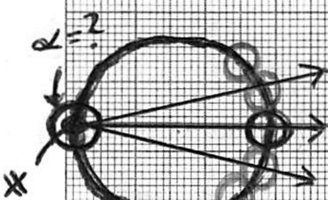
$\varepsilon = \pi/720$

→ Reason: (Extreme case) "When computing α for the center of the unit circle, the 720 different ε-wide strips should still touch / intersect / overlap where they pass through the circle $x^2+y^2=1$."

⇒ The union of all strips covers the entire area inside the circle $x^2+y^2=1$.

$\varepsilon = \pi/360$

→ Other extreme case:



→ point lies on circle (= boundary)

⇒ ε-wide strips MUST TOUCH ON OPPOSITE SIDE OF UNIT CIRCLE!

⇒ $\varepsilon = \pi/360$

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