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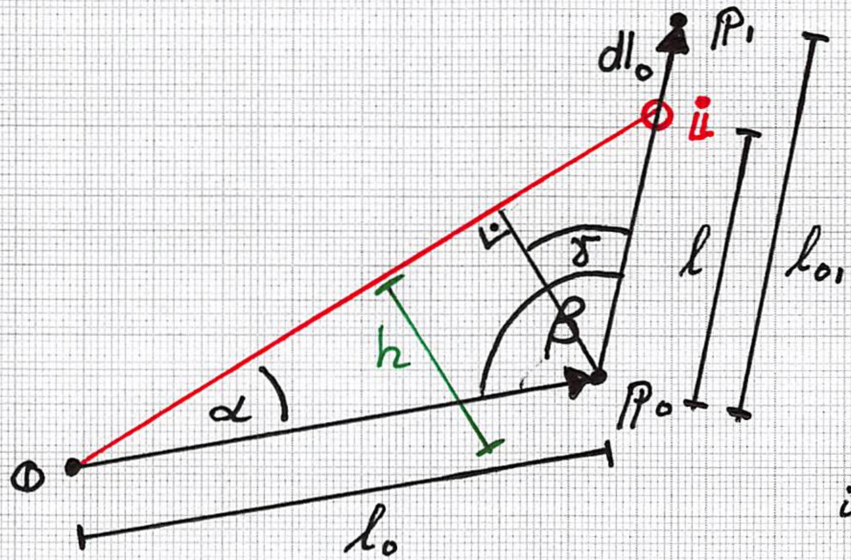
■ ANALYTICAL METHODS – Cont'd.

• Trigonometric computation of edge intersection:



$$i = P_0 + \frac{l}{l_{01}} dl_0$$

i) $c\gamma = \frac{h}{l}$

$\Rightarrow l = \frac{h}{c\gamma}$

ii) $s\alpha = \frac{h}{l_0}$

$\Rightarrow h = l_0 \, s\alpha$

i), ii) $\Rightarrow l = l_0 \frac{s\alpha}{c\alpha}$

iii) $\beta - \gamma + \frac{\pi}{2} + \alpha = \pi \Rightarrow \gamma = \alpha + \beta - \frac{\pi}{2}$

$$\Rightarrow c\gamma = c\left(\alpha + \beta - \frac{\pi}{2}\right) = s(\alpha + \beta)$$

$$\Rightarrow \quad i = P_0 + \frac{l_0}{l_{01}} \cdot \frac{s\alpha}{s(\alpha+\beta)} \, dl_0$$

{ Note: $c\beta = \frac{-P_0 \bullet dl_0}{l_0 \cdot l_{01}}$

$\Rightarrow \beta = \ldots$ }

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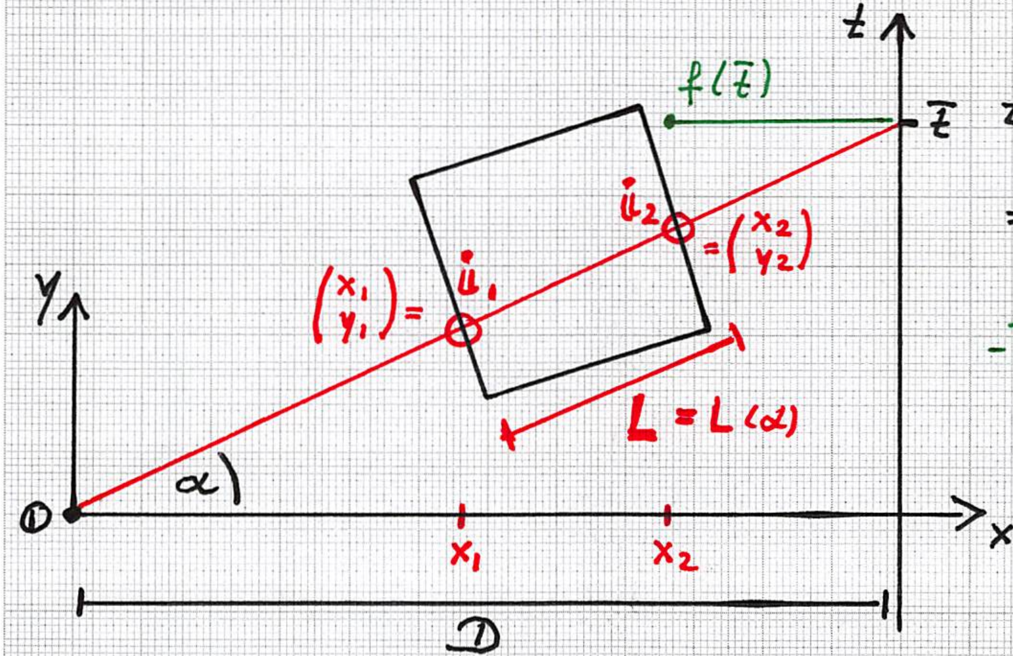
■ ANALYTICAL METHODS – Cont'd.

- More analytical definitions (ray-box intersection)



$$\tan\alpha = \frac{\bar{t}}{D}$$

$$\Rightarrow \underline{\bar{t} = D \tan\alpha}$$

$$f(\bar{t}) = L(\alpha) = f(D\tan\alpha)$$

$$\underline{L(\alpha) = L} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$\left[\begin{array}{l} y_1 = x_1 \tan\alpha, \\ y_2 = x_2 \tan\alpha \end{array} \right]$$

$$= \sqrt{(x_2-x_1)^2 + \tan^2\alpha\,(x_2-x_1)^2}$$

$$= \sqrt{(1+\tan^2\alpha)(x_2-x_1)^2}$$

$$= \underline{\underline{|x_2-x_1| \sqrt{(1+\tan^2\alpha)}}}$$

- **REFERENCES:**
  - → Klaus Müller, dissertation, "Fast and Accurate 3D Reconstruction... Cone-Beam..."
  - → Swan, Möller, ..., Crawfis, Yagel, "Perspective Splatting"
  - → Papers by ALIREZA ENTEZARI, "Box Spline Projection"

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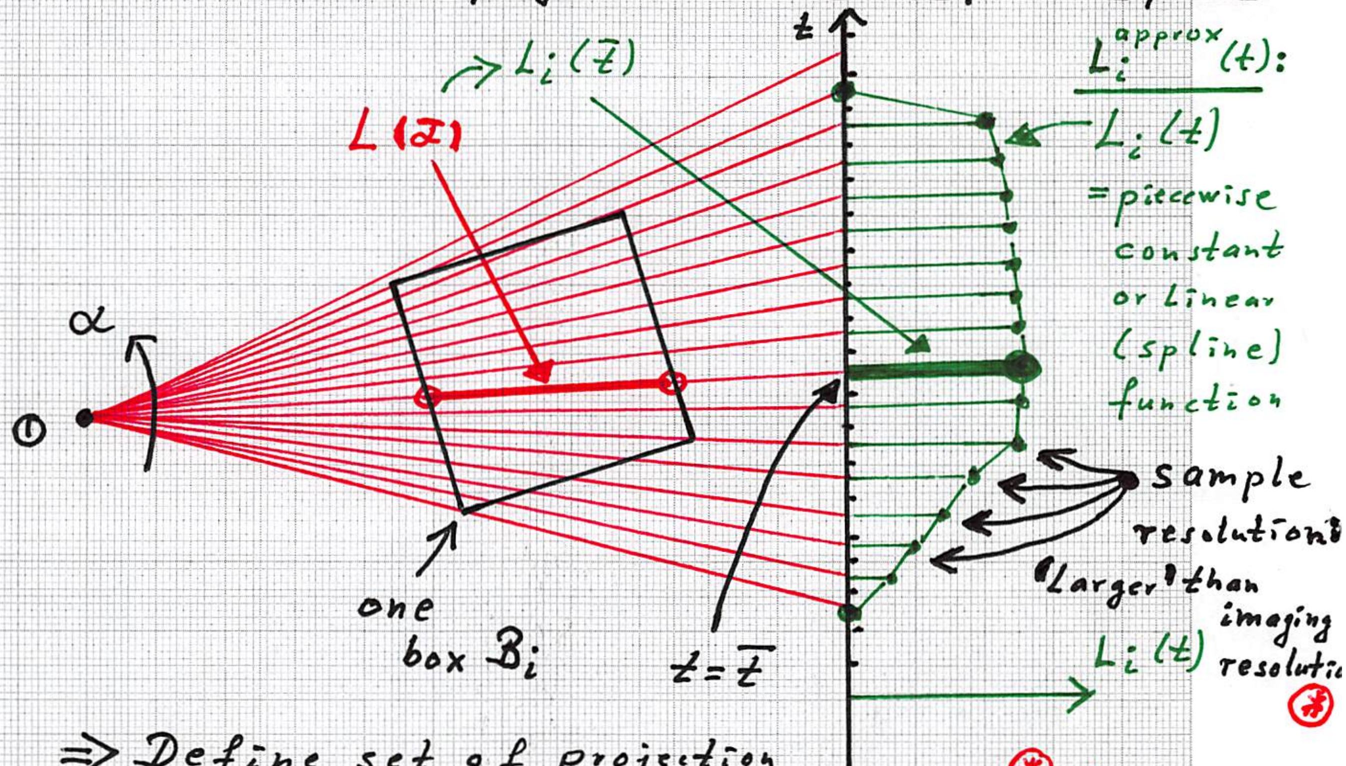
ANALYTICAL METHODS – Cont'd.

- "Analytical alternative" to exact definition of box projections: constant / linear spline



⇒ Define set of projection length functions – SPLINES – $\{L_i(t)\}$ for set of boxes $\{B_i\}$

⊛ 1) Each box must project to an $L_i(t)$ spline

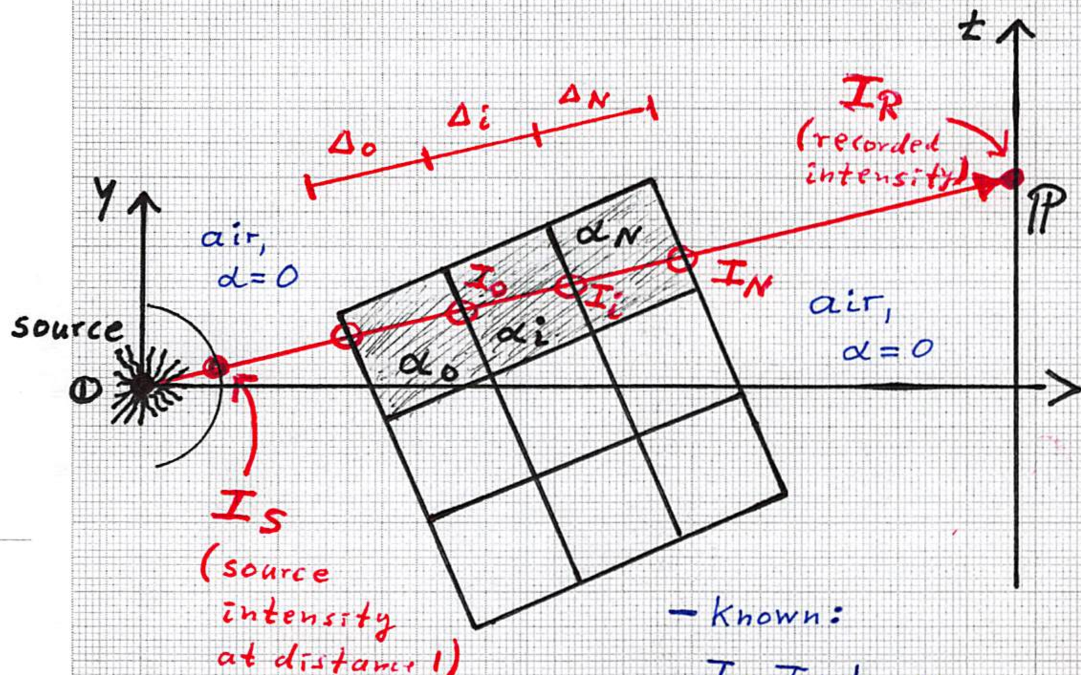
2) Supersampling of the given image pixels must be done to define $L_i(t)$

⇒ Result: ANALYTICAL spline function approximations of the exact projection length function $L_i(t)$!

⇒ Use splines $L_i^{approx}(t)$ for reconstruction!

ANALYTICAL METHODS – Cont'd.

- Point light source + inverse distance law + Cartesian grid of boxes + absorption



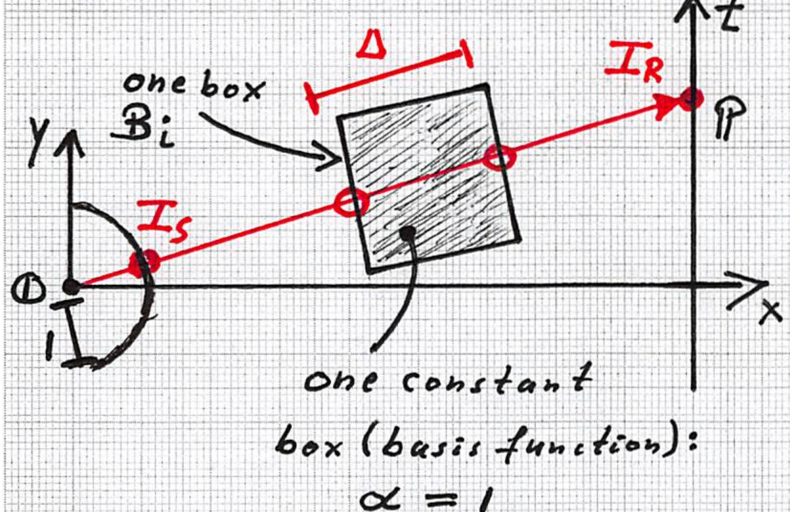
- total length of ray = $L = \|\vec{P}\|$

- Law for 2D case:

$$\boxed{\sum_{i=0}^{N} \Delta_i \alpha_i = -\ln\left(L \frac{I_R}{I_S}\right)}$$

- known: I_S, I_R, L, Δ_i
- compute: α_i

- Recorded/imaged intensity value of one box:



- Law: $\Delta \cdot 1 = -\ln\left(L \frac{I_R}{I_S}\right)$

⇒ recorded image intensity value for ray passing through one box:

$$\boxed{I_R = \frac{I_S}{L} e^{-\Delta}}$$

⇒ Compute analytical form of $I_R(t)$ for every box B_i.

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■ ANALYTICAL METHODS – Cont'd.

- Reconstruction via best approximation:

  - Recorded image intensity data (viewed as a function): $I_R(t)$

  - Set of individual "recorded" box image intensity data (viewed as constant or linear spline functions, for example):

    $I_R^i(t)$, $i = 1 \ldots$ number_of_boxes $(= B)$

  - solve linear system:

!

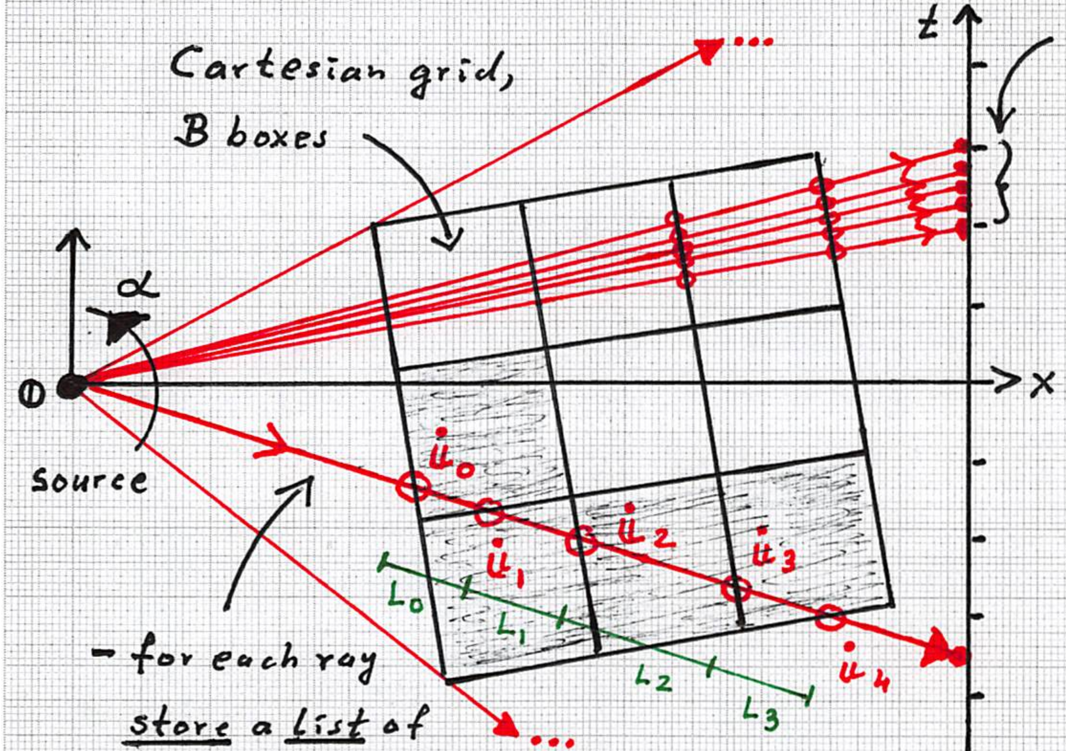
$$\begin{bmatrix} \langle I_R^1, I_R^1 \rangle & \ldots & \langle I_R^1, I_R^B \rangle \\ \vdots & & \vdots \\ \langle I_R^B, I_R^1 \rangle & \ldots & \langle I_R^B, I_R^B \rangle \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_B \end{bmatrix} = \begin{bmatrix} \langle I_R, I_R^1 \rangle \\ \vdots \\ \langle I_R, I_R^B \rangle \end{bmatrix}$$

$\Rightarrow \alpha_1, \ldots, \alpha_B$ are the needed "intensity" / "density" / "opacity" values of the Cartesian grid boxes.

~BH

▪ ANALYTICAL METHODS – Cont'd.

• **Computing and Storing Intersection Points between Rays and Cartesian Grid**



– for each "pixel" on t-line (= projection line) use K uniformly / equidistantly spaced points '•'; these points are the endpoints of all rays (ray segments) starting at O.

– for each ray store a list of intersection points $u_0, u_1, u_2, \ldots$

⇒ Compute for all rays (depending on ray orientation angles α) all intersection points $u_0, u_1, u_2, \ldots$, implying ray segment lengths $L_0, L_1, L_2, \ldots$ This data defines for every box B_i the needed information for a box-specific constant or linear spline function $L_i(t)$ – which is used for best approximation of box density values via least squares. ~ BH