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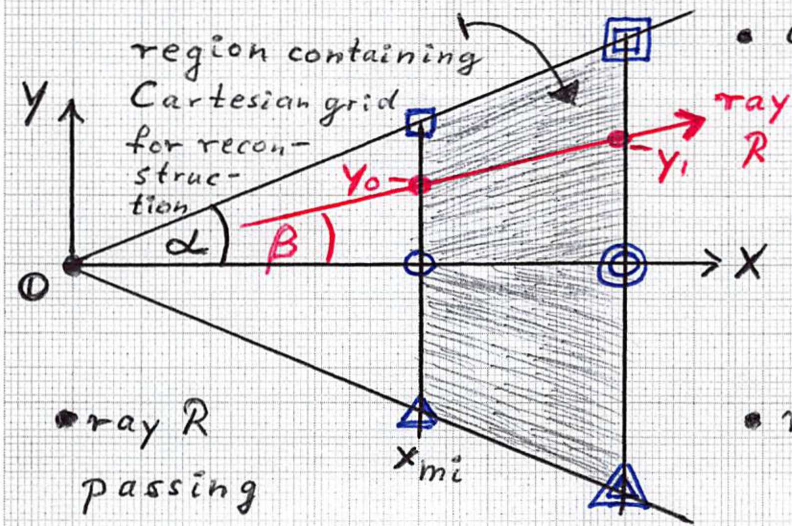
TRANSFORMING PERSPECTIVE TO PARALLEL PROJECTION



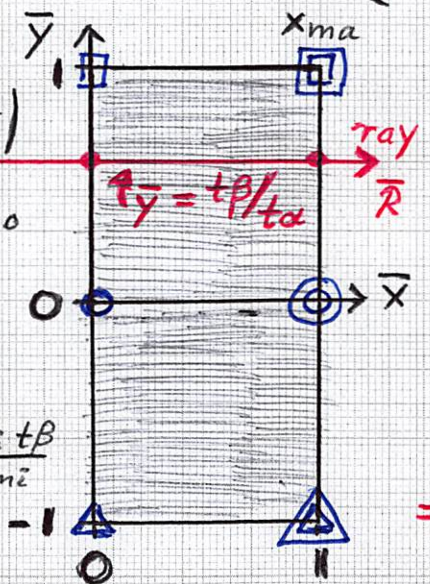
- original region:

$$x_{mi} \le x \le x_{ma}$$
$$-x\,t\alpha \le y \le x\,t\alpha$$
$$(t = \tan)$$

- ray R passing through $\begin{pmatrix} x_{mi} \\ y_0 \end{pmatrix}, \begin{pmatrix} x_{ma} \\ y_1 \end{pmatrix}$

- new region:

$$0 \le \bar{x} \le 1$$
$$-1 \le \bar{y} \le 1$$



- mapping to $\bar{R}$:

$$\underline{y_0} \mapsto \frac{1}{t\alpha}\,\frac{y_0}{x_{mi}} = \frac{1}{t\alpha}\,\frac{x_{mi}\,t\beta}{x_{mi}} = \underline{t\beta / t\alpha}$$

$$\underline{y_1 \mapsto t\beta / t\alpha}$$

- mapping:

$$\boxed{\bar{x} = \frac{x - x_{mi}}{x_{ma} - x_{mi}} = \frac{x - x_{mi}}{\Delta} \qquad \bar{y} = \frac{1}{t\alpha}\cdot\frac{y}{x}\ ,\ x \ne 0}$$

$\Rightarrow$ All rays R emanating from point source O become rays $\bar{R}$ that are parallel to the $\bar{x}$-axis.

- Mapping of selected points:

$$\begin{pmatrix} x_{mi} \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 0 \end{pmatrix},\quad \begin{pmatrix} x_{mi} \\ x_{mi}\,t\alpha \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix},\quad \begin{pmatrix} x_{mi} \\ -x_{mi}\,t\alpha \end{pmatrix} \mapsto \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x_{ma} \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix},\quad \begin{pmatrix} x_{ma} \\ x_{ma}\,t\alpha \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ 1 \end{pmatrix},\quad \begin{pmatrix} x_{ma} \\ -x_{ma}\,t\alpha \end{pmatrix} \mapsto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

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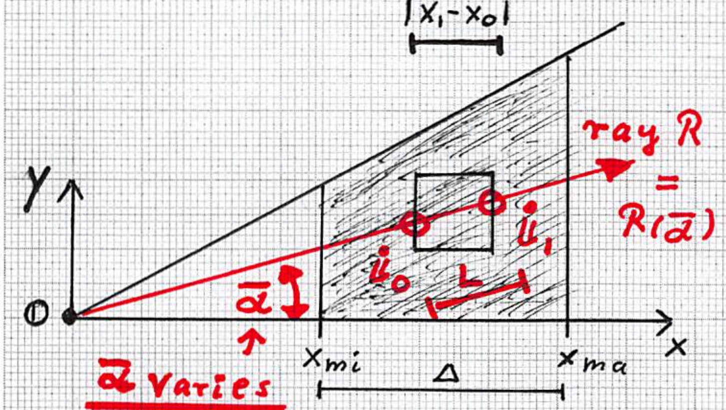
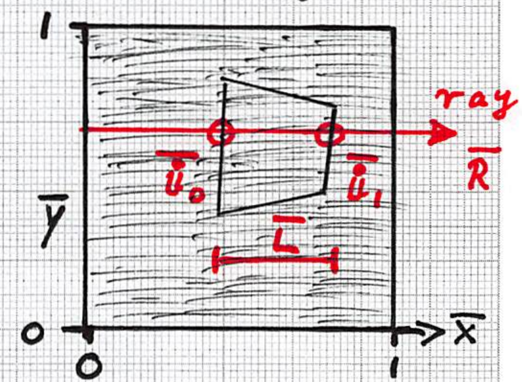
TRANSFORMING PERSPECTIVE TO PARALLEL ...

→ Necessary mapping of ray segment lengths:





- mapping: $\bar{x} = \frac{x - x_{mi}}{\Delta}$, $\bar{y} = \frac{1}{t\bar{\alpha}} \frac{y}{x}$

⇒ squares of Cartesian grid in xy-space

↦ quadrilaterals in $\bar{x}\bar{y}$-space !

- $\bar{u}_0 = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$, $\bar{u}_1 = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$, $\bar{\bar{u}}_0 = \begin{pmatrix} \bar{x}_0 \\ \bar{y}_0 \end{pmatrix}$, $\bar{\bar{u}}_1 = \begin{pmatrix} \bar{x}_1 \\ \bar{y}_1 \end{pmatrix}$

$$\Rightarrow \quad L = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$\bar{L} = \sqrt{(\bar{x}_1 - \bar{x}_0)^2 + (\bar{y}_1 - \bar{y}_0)^2}$$

$$= \sqrt{\left(\frac{x_1 - x_{mi} - x_0 + x_{mi}}{\Delta}\right)^2 + \left(\frac{1}{t\bar{\alpha}}\left(\frac{y_1}{x_1} - \frac{y_0}{x_0}\right)\right)^2}$$

$$= \sqrt{\frac{(x_1 - x_0)^2}{\Delta^2} + \frac{1}{t^2\bar{\alpha}}\underbrace{\left(\frac{y_1}{x_1} - \frac{y_0}{x_0}\right)^2}_{=0}} = \frac{|x_1 - x_0|}{\Delta}$$

⇒ scaling factor: $s = \bar{L} / L$

⇒ Needed are length values L in Cartesian grid: $L = \bar{L} / s$ (**NOTE:** $\bar{\alpha}$ varies, but s is constant; s independent of $\bar{\alpha}$)

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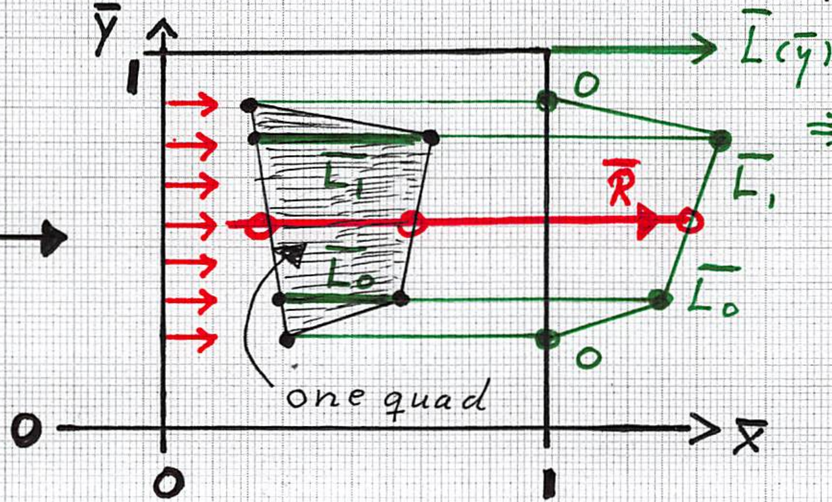
TRANSFORMING: PERSPECTIVE TO PARALLEL...

→ Function $\bar{L}(\bar{y})$ describing ray segment lengths in $\bar{x}\bar{y}$-space:



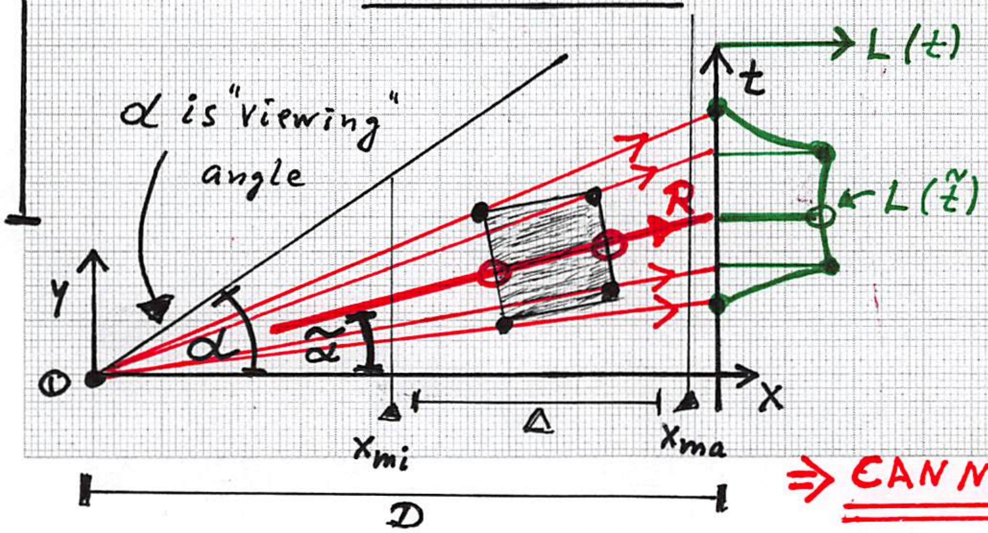
⇒ $\bar{L}(\bar{y})$ is a piecewise linear function (linear spline), in $\bar{x}\bar{y}$-space.

$\bar{x} = \dfrac{x - x_{mi}}{\Delta}$

$\bar{y} = \dfrac{1}{t\alpha}\,\dfrac{y}{x}$

⇒ The exact analytically defined ray segment length function $\bar{L}(\bar{y})$ must be mapped back to the original xy-space.

⇒ ONE OBTAINS AN EXACT ANALYTICAL DEFINITION OF RAY SEGMENT LENGTHS IN SQUARES OF ORIGINAL CARTESIAN GRID!



• specific ray R:
$\tan\tilde{\alpha} = \tilde{t}/D$
$\Rightarrow \tilde{t} = D\tan\tilde{\alpha}$,
$\tilde{\alpha} = \tan^{-1}(\tilde{t}/D)$

⇒ CAN NOW MAP $\bar{L}(\bar{y})$ TO $L(t)$

~BH