Bernd Hamann 5/15/2020

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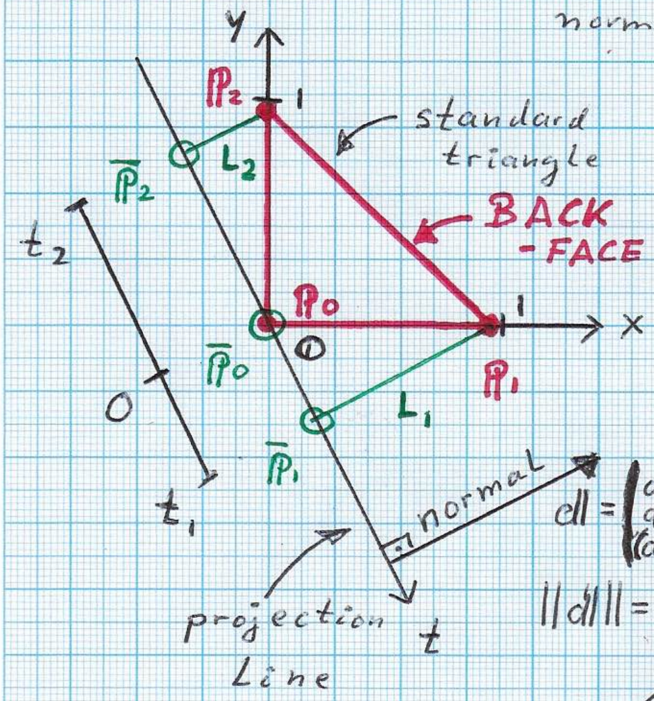
RECONSTRUCTION: ANALYTICAL PROJECTION OF STANDARD SIMPLICES

→ **Idea:** Compute distances of simplex faces from projection plane

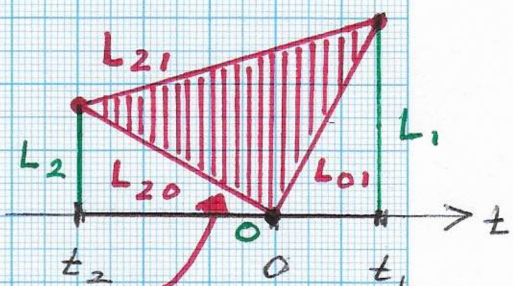
• 2D triangle case: Projection line with specific normal passing through origin



$\mathbb{d} = \begin{pmatrix} d_x \\ d_y \\ (d_z) \end{pmatrix}$

$\|\mathbb{d}\| = 1$

⇒ L-function is a piecewise linear function over t:



$$L(t) = \begin{cases} |L_{21}(t) - L_{20}(t)|, & t \in [t_2, 0] \\ |L_{21}(t) - L_{01}(t)|, & t \in [0, t_1] \end{cases}$$

Where:

$L_{20}(t) = \frac{L_2}{t_2} t, \quad t \in [t_2, 0]$

$L_{01}(t) = \frac{L_1}{t_1} t, \quad t \in [0, t_1]$

$L_{21}(t) = \frac{t_1 - t}{t_1 - t_2} L_2 + \frac{t - t_2}{t_1 - t_2} L_1,$

$t \in [t_2, t_1]$

⇒ The "shadow"/"x-ray" of the standard triangle is based on an analytical definition of the needed L-function.

• L-function defined by distance differences of BACK-FACE and two FRONT-FACES of triangle relative to projection line!

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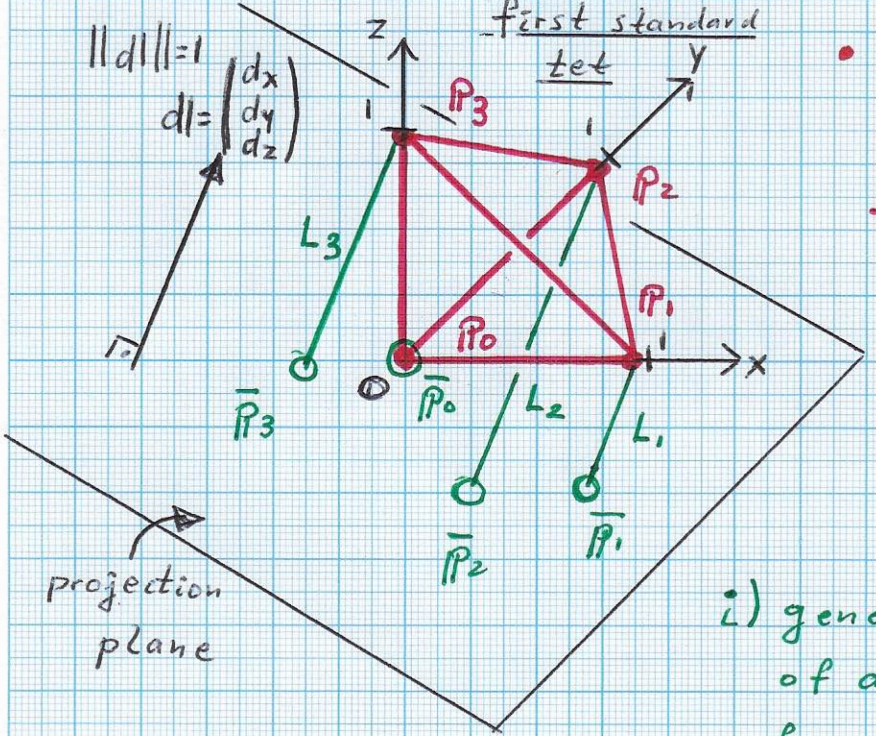
RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

- 3D tet case: Projection plane passing through origin

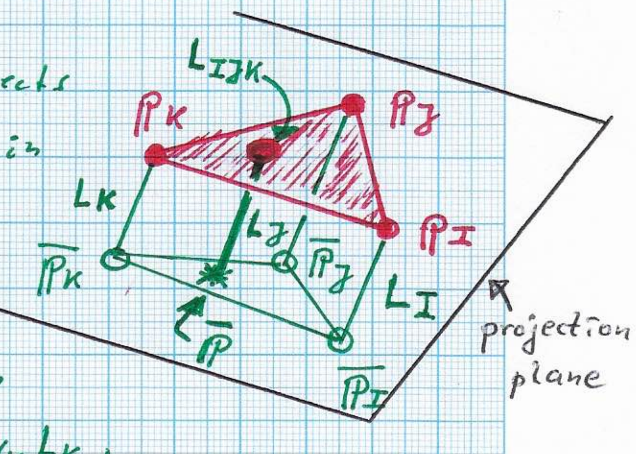


- L-function defined by perpendicular distances of tet faces from projection plane; certain differences of these distances define L-function analytically.

i) general rule: distance L of any point $\mathbb{p} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ from plane with normal $\mathbb{d}$:

$$L = |\langle \mathbb{p}, \mathbb{d} \rangle|$$

ii) Face $\mathbb{p}_J \mathbb{p}_K \mathbb{p}_L$ (of tet), $I, J, K \in \{0,1,2,3\}$ and $I \neq J$, $I \neq K$, $J \neq K$, projects to triangle $\overline{\mathbb{p}_I}\,\overline{\mathbb{p}_J}\,\overline{\mathbb{p}_K}$ in projection plane.



iii) Each of the 4 tet faces defines a linear function

! $L_{IJK}(\mathbb{u}) = u_I L_I + v_J L_J + w_K L_K$,

where $\mathbb{u} = (u_I, v_J, w_K)$ is the barycentric coord. tuple of $\overline{\mathbb{p}}$, relative to $\overline{\mathbb{p}_I}, \overline{\mathbb{p}_J}, \overline{\mathbb{p}_K}$.

iv) Differences of BACK-FACING and FRONT-FACING triangle faces define piecewise linear L-fct.

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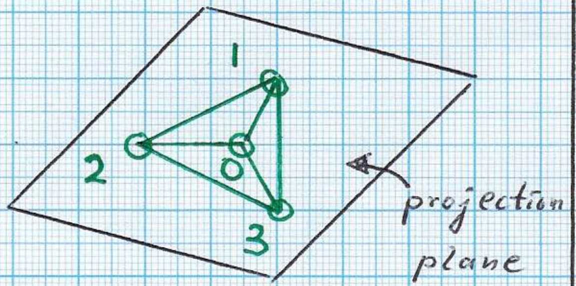
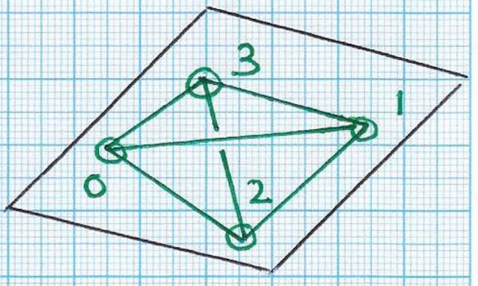
▪ RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

v) Two cases must be considered:

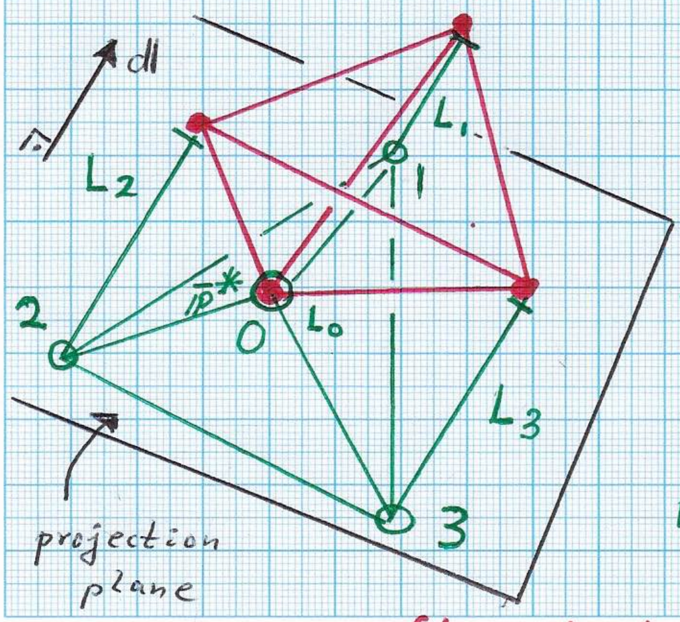
Case I: tet projects to tri | Case II: tet projects to quad





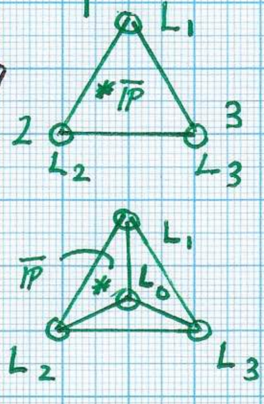
⇒ must handle both cases when computing **piecewise linear L-function** via differences of linear "tet face distance functions" (= perpendicular distances between tet faces and projection plane)

• Case I



• Four linear functions, one per projected face:

$L_{012}, L_{013}, L_{023}, L_{123}$



$L_{123} = \text{Lin Int}(L_1, L_2, L_3)$

$L_{012} = \text{Lin Int}(L_0, L_1, L_2)$

$L_{013} = \text{Lin Int}(L_0, L_1, L_3)$

$L_{023} = \text{Lin Int}(L_0, L_2, L_3)$

$$\Rightarrow L(\overline{p}) = \begin{cases} |L_{123} - L_{012}|, & \overline{p} \in \Delta_{012} \\ |L_{123} - L_{013}|, & \overline{p} \in \Delta_{013} \\ |L_{123} - L_{023}|, & \overline{p} \in \Delta_{023} \end{cases} \quad !$$

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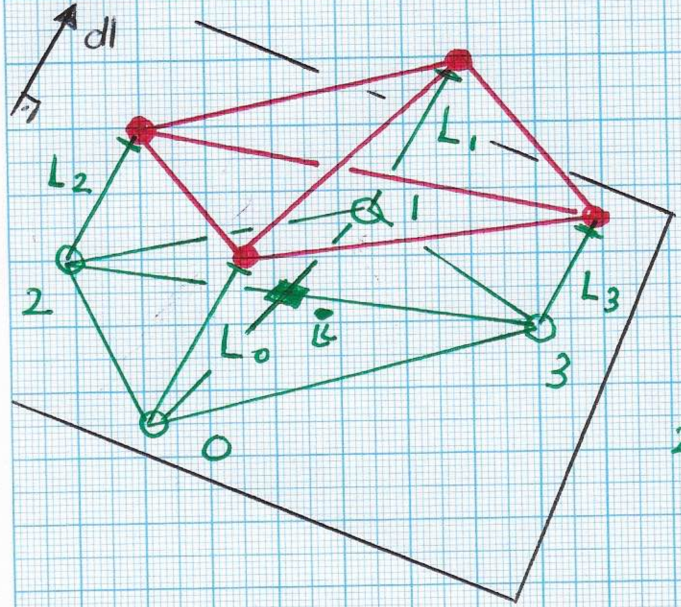
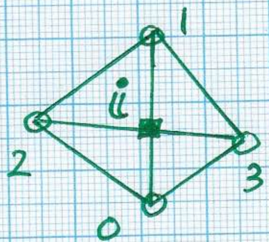
- RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

{ NOTE: Special sub-case of Case I: two tet vertices project to the same point in projection plane!
⇒ L-function has only ONE linear piece. }

- Case II



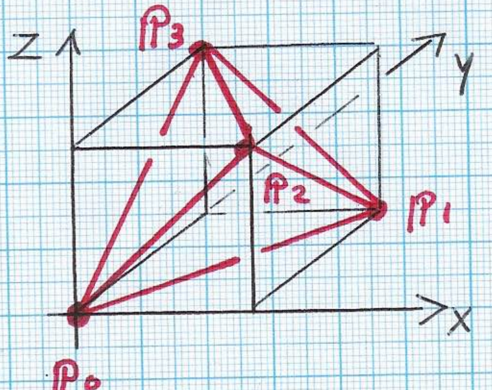
- Four linear distance functions as in Case I: L_{012}, L_{013}, L_{023}, L_{123}
- Line segments $\overline{01}$ and $\overline{23}$ intersect in point ũ.



⇒ L-function is piecewise linear function with four pieces.

- The four linear pieces are defined over 02ũ, 03ũ, 12ũ, 13ũ.

- The second standard tet (all edges having length $\sqrt{2}$)



⇒ L-function is defined as a piecewise linear function over a triangular or quadrangular domain in the projection plane. Again, the domain consists of triangular sub-domains.

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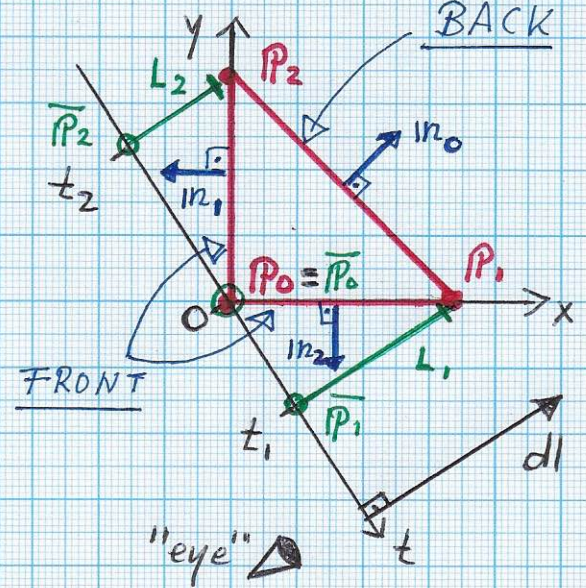
RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

→ Use of FRONT- and BACK-FACE normals

• 2D case



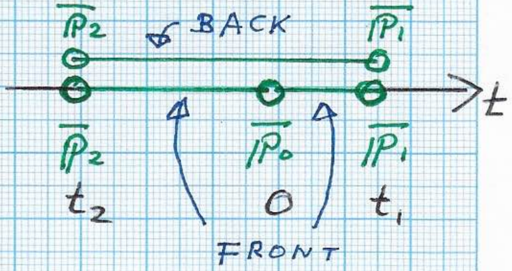
• $\mathbb{n}_i$ = UNIT OUTWARD normal of a face

• Test:

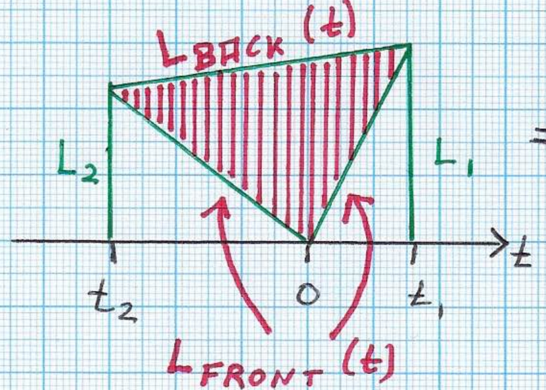
$\mathbb{d} \bullet \mathbb{n}_i < 0 \Rightarrow$ FRONT face

$\mathbb{d} \bullet \mathbb{n}_i > 0 \Rightarrow$ BACK face

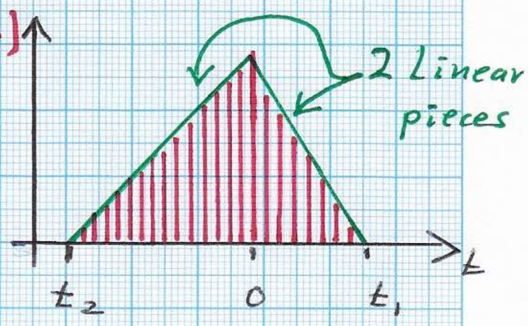
• Faces (= edges) projected onto t-Line are FRONT or BACK faces:



⇒ **L-function computation:**



⇒

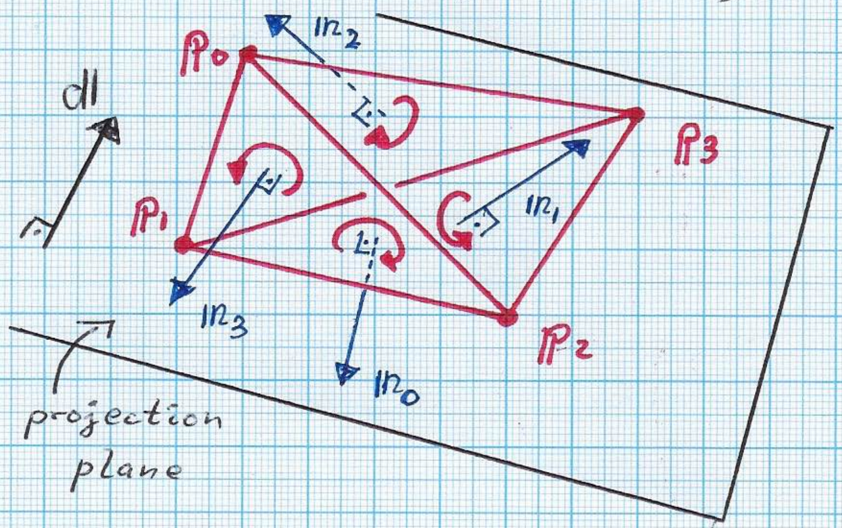


$L(t) = L_{BACK}(t) - L_{FRONT}(t)$ **!**

• One can use OUTWARD tet face normals to determine FRONT and BACK faces. To obtain the L-function one computes the difference of the distances of the back and front faces (from projection plane).

RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

• 3D Case (FRONT, BACK faces)

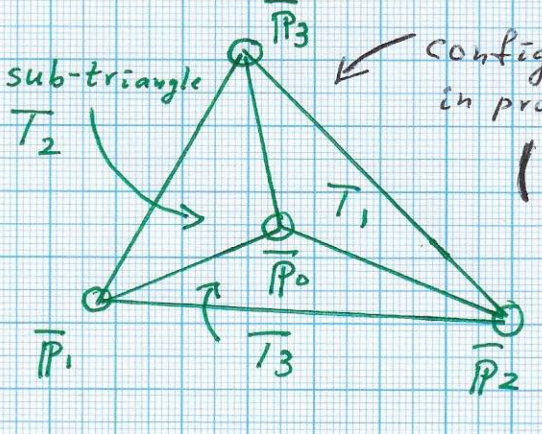


• Test:
$\mathbb{d} \cdot \mathbb{n}_i < 0 \Rightarrow$ FRONT face
$\mathbb{d} \cdot \mathbb{n}_i > 0 \Rightarrow$ BACK face

• All unit face normals are OUTWARD normals.

⇒ **L-function computation:**

→ Ex: Tet projects to triangular region



(degenerate case: two tet corners projecting to same point)

• L-function = piecewise linear fct. with three linear pieces defined over triangular subdomains ($\overline{\mathbb{P}}_0\overline{\mathbb{P}}_1\overline{\mathbb{P}}_2$, $\overline{\mathbb{P}}_0\overline{\mathbb{P}}_2\overline{\mathbb{P}}_3$, $\overline{\mathbb{P}}_0\overline{\mathbb{P}}_3\overline{\mathbb{P}}_1$)

→ Ex: Face $\mathbb{P}_1\mathbb{P}_2\mathbb{P}_3$ is FRONT face, other faces are BACK faces

$$\Rightarrow \underline{L} = \begin{cases} LinInt(L_0, L_1, L_2) - b_{\#} & \text{in } T_3 \\ LinInt(L_0, L_2, L_3) - b_{\#} & \text{in } T_1 \\ LinInt(L_0, L_3, L_1) - b_{\#} & \text{in } T_2 \end{cases}$$

where $b = LinInt(L_1, L_2, L_3)$

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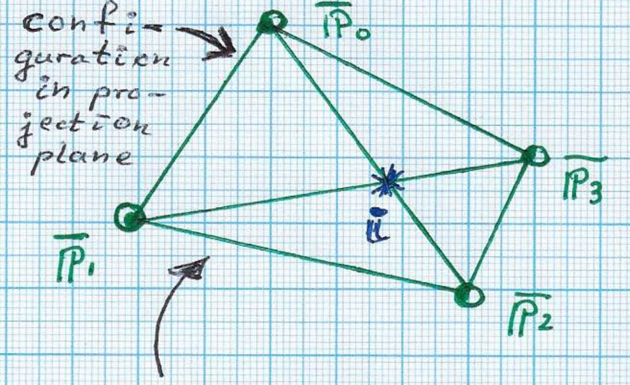
■ RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

... L-function computation:

➔ Ex: Tet projects to quadrangular region



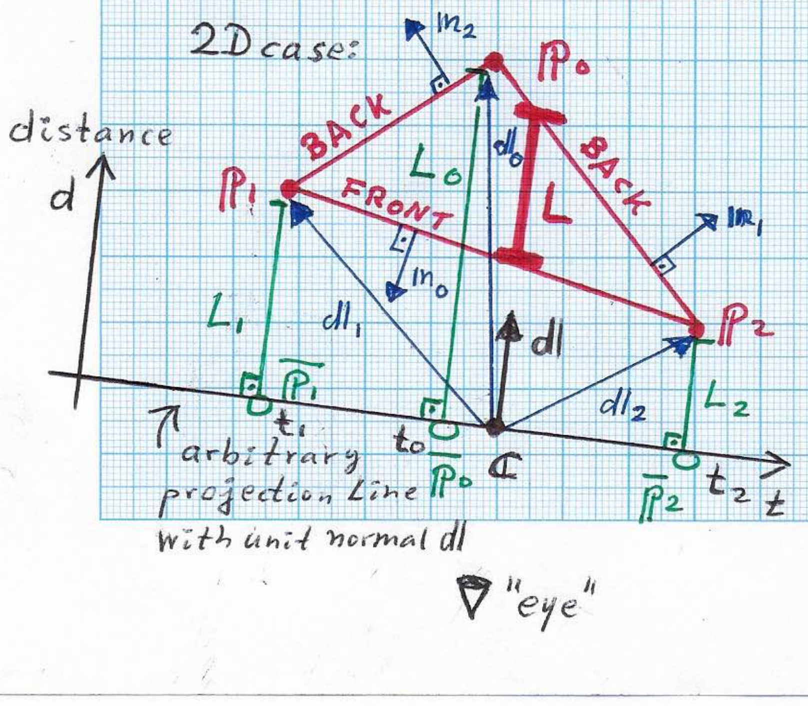
- L-function = piecewise linear function with four liner pieces defined over triangular sub-domains ($u\overline{P}_0\overline{P}_1$, $u\overline{P}_1\overline{P}_2$, $u\overline{P}_2\overline{P}_3$, $u\overline{P}_3\overline{P}_0$)

possibility: faces $P_0P_1P_2$ and $P_0P_2P_3$ BACK faces and faces $P_0P_1P_3$ and $P_1P_2P_3$ FRONT faces

⇒ **L =** "difference of distance functions, restricted to the respective four sub-domains"

! • GENERALIZATION OF L-VALUE COMPUTATION FOR PERSPECTIVE PROJECTIO, ARBITRARY PROJ. PLANE

2D case:



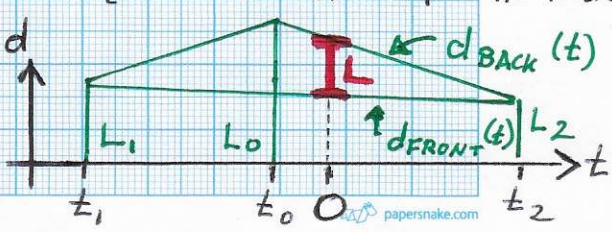
C = center of projection

$d_i = P_i - C$

$L_i = d \cdot d_i$ (dot product)

$\overline{P}_i = P_i - L_i d$

n_i = unit outward face normal



$L = L(0) = L_{BACK}(0) - L_{FRONT}(0)$

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