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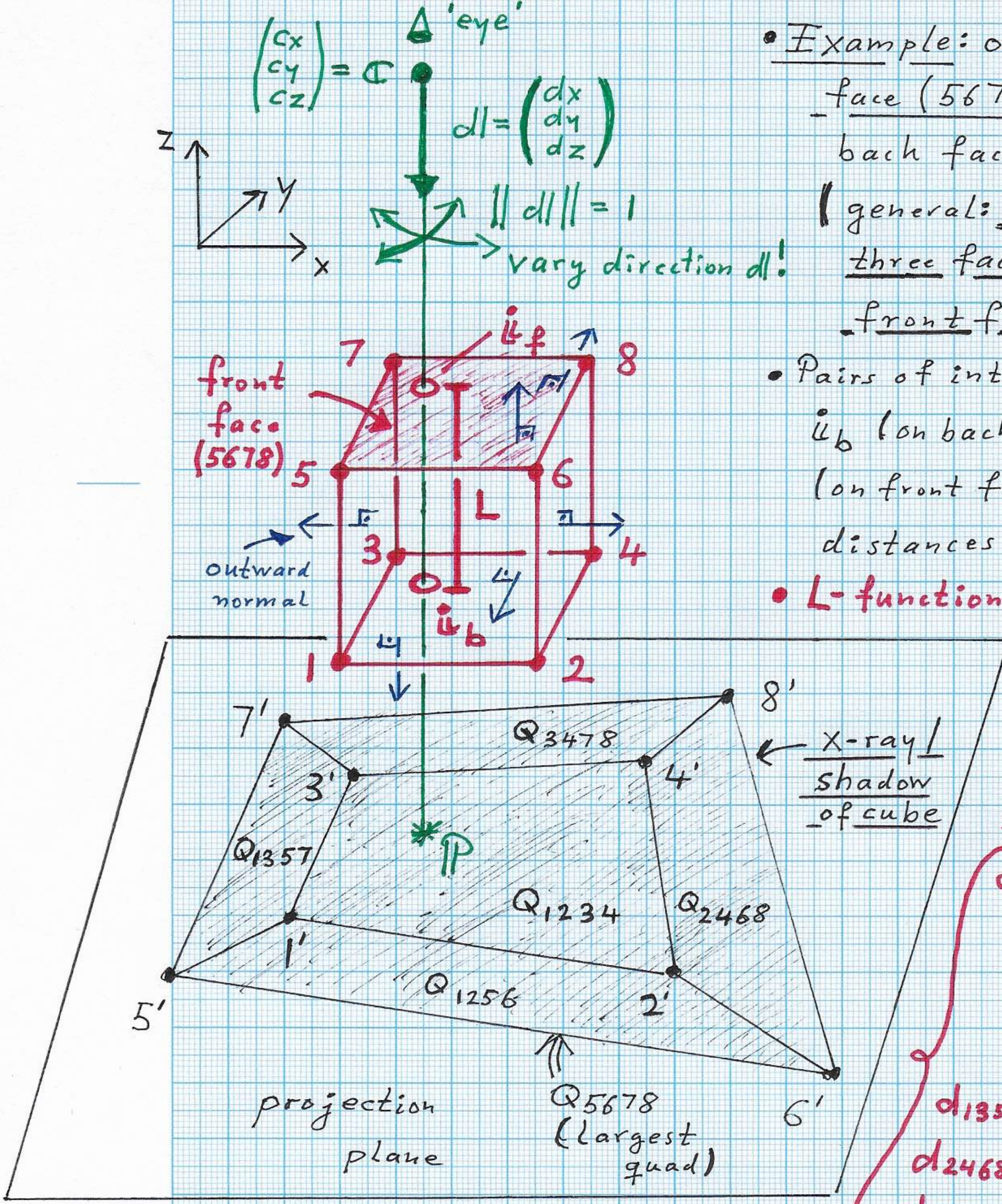
RECONSTRUCTION: ANALYTICAL PROJECTION - Cont'd.

... 2) ii) 3D Case: Cube, Perspective Projection



- Example: only one front face (5678) and five back faces (general: one, two or three faces can be front faces...)
- Pairs of intersection points i_b (on back face) and i_f (on front face) define distances relative to $\mathbb{C}$
- L-function defined for points $\mathbb{P}$ in projection plane:

$$\underline{\underline{L(\mathbb{P})}} = \begin{cases} d_{1234} - d_{5678} & (\text{in } Q_{1234}), \\ d_{1256} - d_{5678} & (\text{in } Q_{1256}), \\ d_{1357} - d_{5678} & (\text{in } Q_{1357}), \\ d_{2468} - d_{5678} & (\text{in } Q_{2468}), \\ d_{3478} - d_{5678} & (\text{in } Q_{3478}) \end{cases}$$

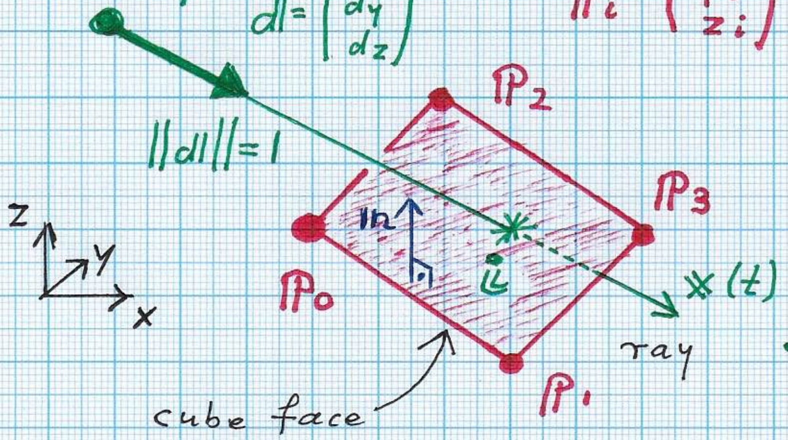
- distance d_{ijkl} = distance between center $\mathbb{C}$ and intersection point of ray and cube face $(ijkl)$

RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

... 2) ii) 3D Case: Cube, Perspective Projection

→ Must compute intersection point between ray and cube face:

$$\mathbb{C} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix} \quad \mathbb{d} = \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix} \quad \mathbb{P}_i = \begin{pmatrix} x_i \\ y_i \\ z_i \end{pmatrix} \quad \mathbb{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$$



$$\mathbb{n} = \frac{(\mathbb{P}_1 - \mathbb{P}_0) \times (\mathbb{P}_2 - \mathbb{P}_0)}{\|(\mathbb{P}_1 - \mathbb{P}_0) \times (\mathbb{P}_2 - \mathbb{P}_0)\|}$$

(= unit normal of face)

- Compute point $\mathbb{i}$ by intersecting PARAMETRIC ray and IMPLICIT plane (containing the cube face):

$$\mathbb{i} = \mathbb{C} + t_i \mathbb{d}$$

(i) RAY: $\mathbb{x}(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \mathbb{C} + t\mathbb{d}$

(ii) PLANE: $\mathbb{n} \bullet (\mathbb{x} - \mathbb{P}_0) = 0$

(difference vector of point $\mathbb{x}$ and $\mathbb{P}_0$ perpendicular to $\mathbb{n}$)

- Insert (i) into (ii): $\begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} \bullet \begin{pmatrix} c_x + t d_x - x_0 \\ c_y + t d_y - y_0 \\ c_z + t d_z - z_0 \end{pmatrix} = 0$

$\hookrightarrow \mathbb{n} \bullet \mathbb{C} + t \cdot \mathbb{n} \bullet \mathbb{d} - \mathbb{n} \bullet \mathbb{x}_0 = 0$

$\hookrightarrow t \cdot \mathbb{n} \bullet \mathbb{d} = \mathbb{n} \bullet \mathbb{x}_0 - \mathbb{n} \bullet \mathbb{C}$

$\hookrightarrow (t_i =)\, t = \dfrac{\mathbb{n} \bullet (\mathbb{x}_0 - \mathbb{C})}{\mathbb{n} \bullet \mathbb{d}}$

(Note: $\mathbb{i}$ is inside face $\mathbb{P}_0 \mathbb{P}_1 \mathbb{P}_2 \mathbb{P}_3 \Leftrightarrow$ $(\mathbb{i} - \mathbb{P}_0) = u(\mathbb{P}_1 - \mathbb{P}_0) + v(\mathbb{P}_2 - \mathbb{P}_0)$, where $u, v \in [0, 1]$)

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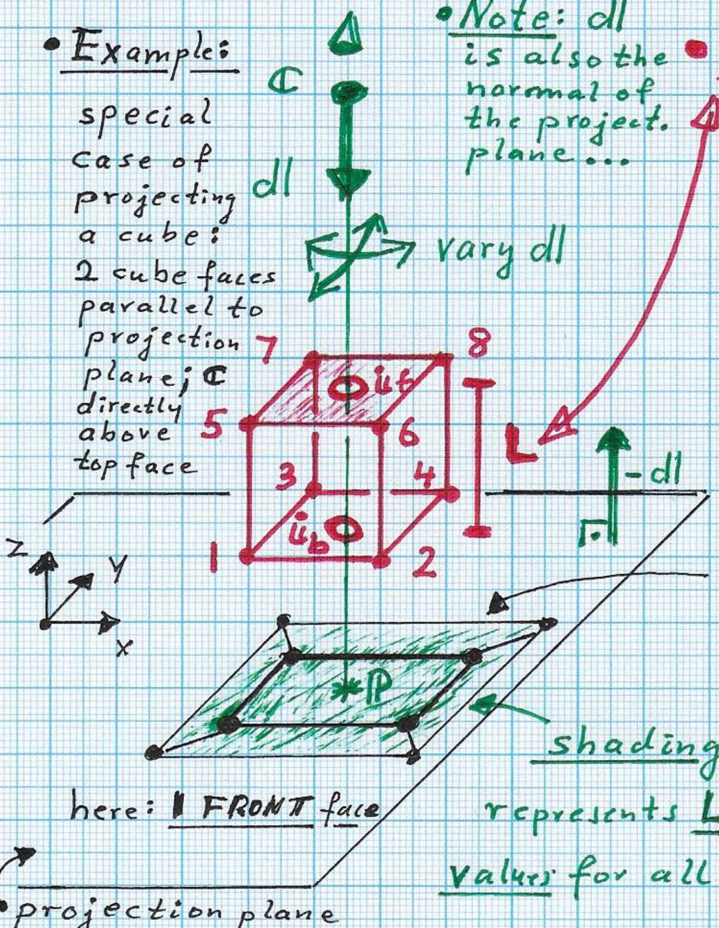
▪ RECONSTRUCTION: ANALYTICAL PROJECTION – Cont'd.

... 2) ii) 3D Case: Cube, Perspective Projection

• Example: special case of projecting a cube: 2 cube faces parallel to projection plane; $\mathbb{C}$ directly above top face

• Note: dl is also the normal of the project. plane...



here: ▮ FRONT face

projection plane

shading represents L values for all rays passing through cube

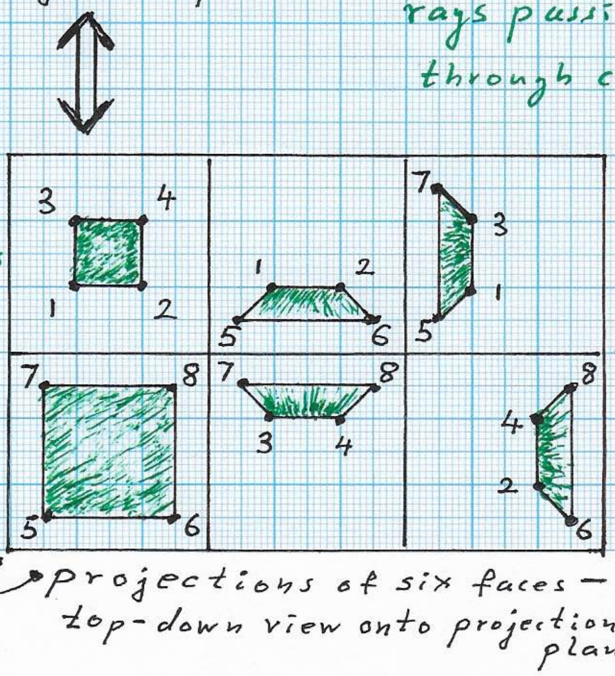
boundary of x-ray/shadow of cube defined by boundary of convex hull of eight projected cube corners

• L-function generation: Shooting bundle of rays, emanating from $\mathbb{C}$, through cube; distance between "entry" and "exit" points defines L-value for point $\mathbb{P}$ in projection plane

• L-function parametrization: can use $L = L$ (2D projection plane) or $L = L$ (long.–lat. spherical coords.)

(dl can be parametrized using spherical coords.)

• distance functions for each of the 6 cube faces $ijkl$: $d_{i,j,k,l}$



projections of six faces – top-down view onto projection plane

shading represents distances between $\mathbb{C}$ and faces $ijkl$;

$L = \| u_b - u_f \|$ = distance of BACK and FRONT FACE points