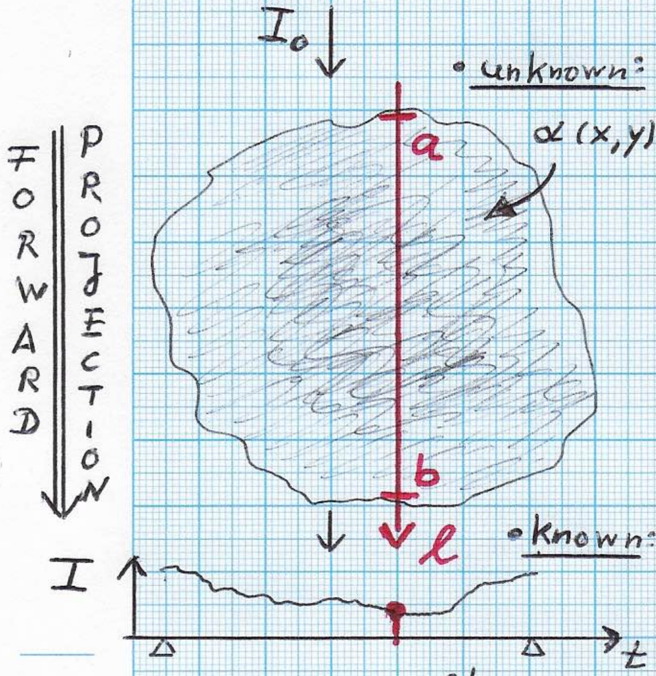
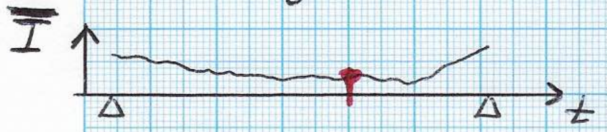
Stratovan

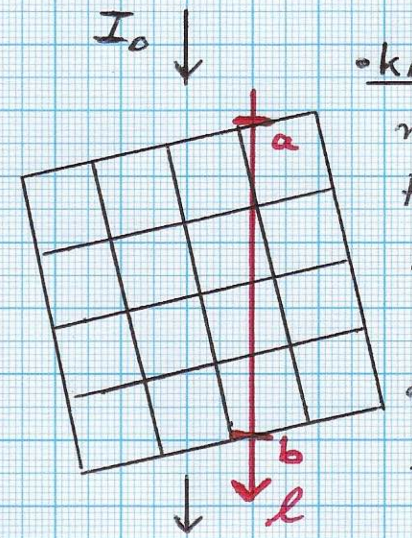
## ■ WAVELET RECONSTRUCTION – Cont'd.




$$I = I_0\, e^{-\int_a^b \alpha(\ell)\, d\tilde{\ell}}$$



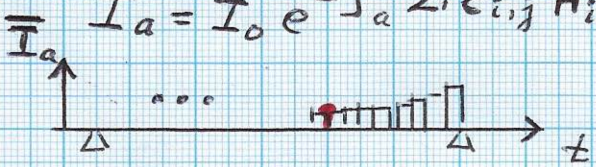

$$\bar{I} = \ln I = $$
$$= \ln I_0 - \int_a^b \alpha(\ell)\, d\tilde{\ell}$$




• known: normalized Haar wavelet basis functions, $\{H_{i,j}(x,y)\}$

• unknown: BEST expansion $\alpha^a(x,y) = \sum c_{i,j} H_{i,j}$

$$I_a = I_0\, e^{-\int_a^b \sum c_{i,j} H_{i,j}(\ell)\, d\tilde{\ell}}$$




$$\bar{I}_a = \ln I_a =$$
$$= \ln I_0 - \int_a^b \sum c_{i,j} H_{i,j}(\ell)\, d\tilde{\ell}$$
$$= \ln I_0 - \sum c_{i,j} \int_a^b H_{i,j}(\ell)\, d\tilde{\ell}$$

• **Goal:** Minimize integrated squared difference between $\bar{I}$ and $\bar{I}_a$ $\Rightarrow$ definition of best least squares approximation of $\alpha(x,y)$

$\Rightarrow$ "IDEAL": $\bar{I}_a = \bar{I}$

$$\Rightarrow \sum_{i,j} c_{i,j} \int_a^b H_{i,j}(\ell)\, d\tilde{\ell} = \ln I$$

(considering entire t-value range)

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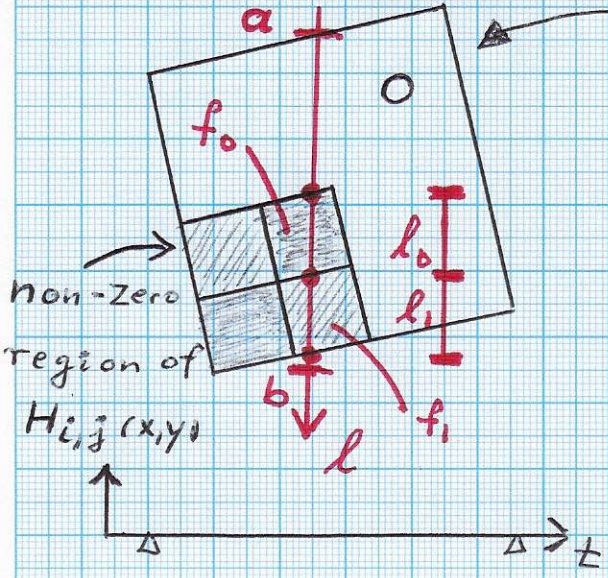
## ▪ WAVELET RECONSTRUCTION – Cont'd.

- **Note:** Meaning of $\int_a^b H_{i,j}(\ell)\, d\tilde{\ell}$




example:

$$\int_a^b H_{i,j}(\ell)\, d\tilde{\ell} =$$

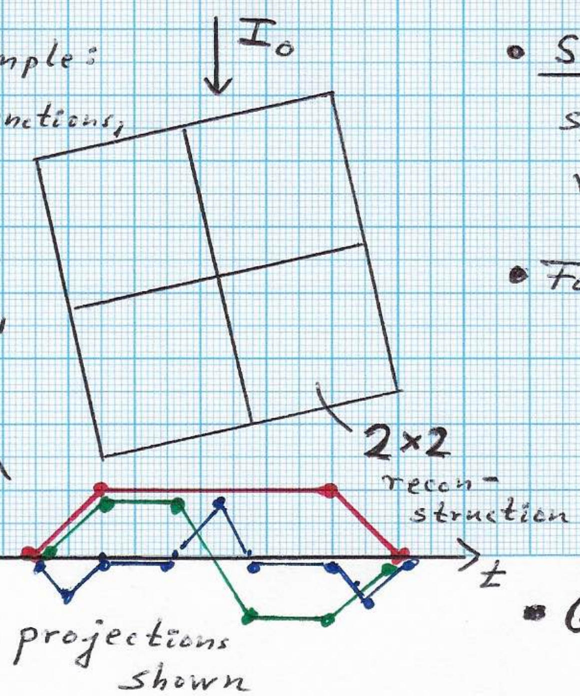
$$= \ell_0 f_0 + \ell_1 f_1$$

$$\underset{\text{general}}{=} \sum_{k=0}^{K-1} \ell_k f_k$$

($\ell_k$ = ray segment length
$f_k$ = value of basis function)

$\Rightarrow$ "IDEAL": $\displaystyle\sum_{i,j} c_{i,j} \left( \sum_{k=0}^{K_{i,j}-1} \ell_k f_k \right) = \ln I$

- Example:
  4 functions,
  $H_{0,0}$
  $H_{1,0}$
  $H_{0,1}$
  $H_{1,1}$




- Solution: BEST least squares approximation via normal equations

- Forward projection of $H_{i,j}$:

$$\bar{I}_{i,j}(t) = \ln I_0 - \left( \int_a^b H_{i,j}(\ell)\, d\tilde{\ell} \right)(t)$$

- Given: $\bar{I}(t) = \ln I(t)$

## WAVELET RECONSTRUCTION – Cont'd.

- Resulting least squares BEST approximation

i) Function (usually given in discretized form) to be approximatet:

$$\overline{I} = \overline{I}(t) = \ln I(t)$$

ii) Basis functions used for best approximation:

$$\overline{I}_{i,j} = \overline{I}_{i,j}(t) = \ln I_0 - \left( \int_{\tilde{\ell}=a}^{b} H_{i,j}(\ell)\, d\tilde{\ell} \right)(t)$$

setting $I_0 = 1$ (when possible)
$\Rightarrow \ln I_0 = 0$

iii) Normal equations:

- example: 16 basis functions $\overline{I}_{0,0} \ldots \overline{I}_{3,3}$

$$\begin{bmatrix} \langle \overline{I}_{00}, \overline{I}_{00} \rangle & \langle \overline{I}_{00}, \overline{I}_{10} \rangle & \cdots & \langle \overline{I}_{00}, \overline{I}_{33} \rangle \\ \langle \overline{I}_{10}, \overline{I}_{00} \rangle & \langle \overline{I}_{10}, \overline{I}_{10} \rangle & \cdots & \langle \overline{I}_{10}, \overline{I}_{33} \rangle \\ \vdots & & & \vdots \\ \langle \overline{I}_{33}, \overline{I}_{00} \rangle & \langle \overline{I}_{33}, \overline{I}_{10} \rangle & \cdots & \langle \overline{I}_{33}, \overline{I}_{33} \rangle \end{bmatrix} \mathbb{C} = \begin{bmatrix} \langle \overline{I}, \overline{I}_{00} \rangle \\ \langle \overline{I}, \overline{I}_{10} \rangle \\ \vdots \\ \langle \overline{I}, \overline{I}_{33} \rangle \end{bmatrix}$$

$\Rightarrow$ needed coefficient vector $\mathbb{C} = (c_{00}, \ldots, c_{33})^T$

iv) BEST approximation of reconstruction $\alpha(x,y)$:

$$\alpha^a(x,y) = \sum_{i=0}^{3} \sum_{j=0}^{3} c_{i,j} H_{i,j}(x,y)$$



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$\Rightarrow$ multi-res./scale-specific features, analysis, classification!

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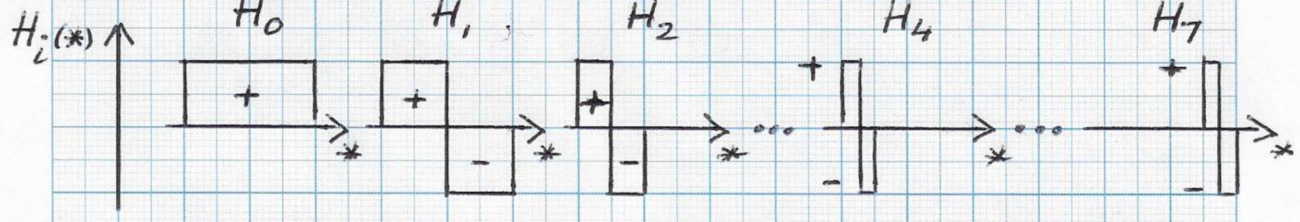
## ▪ WAVELET RECONSTRUCTION – Cont'd.

• Multi-scale analysis via "wavelet scale features"

Example: 8-by-8 Haar wavelet reconstruction

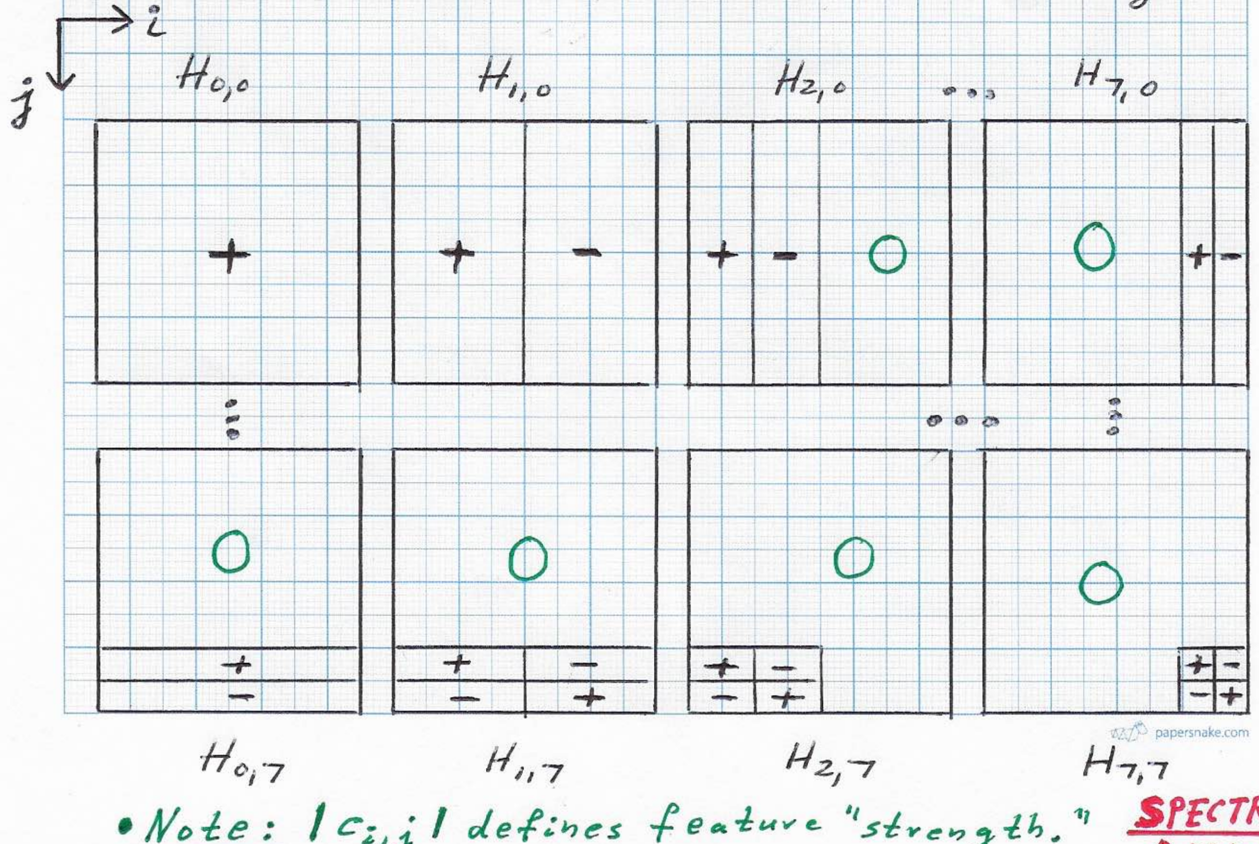



⇒ Bivariate Haar wavelet basis functions defined as

$$H_{i,j}(x,y) = H_i(x)\,H_j(y) \quad , \quad i,j = 0 \ldots 7$$

plus normalization: $H_{i,j} = H_{i,j} \big/ \sqrt{\langle H_{i,j}, H_{i,j} \rangle}$

• "Portraits" and associated coefficients $c_{i,j}$ of $H_{i,j}$ define features and feature "strengths"

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• Note: $|c_{i,j}|$ defines feature "strength."

SPECTRAL ANALYSIS !