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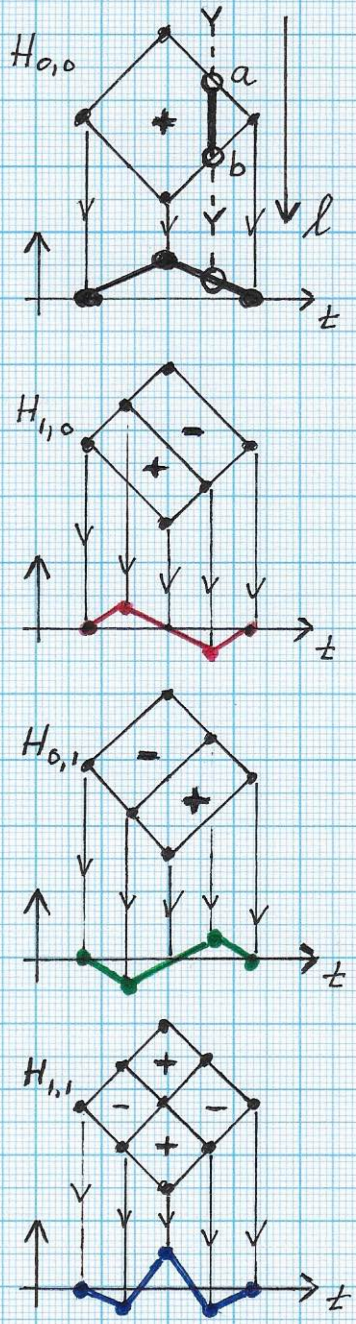
7/6/2020
Stratovan

▪ WAVELET RECONSTRUCTION - Cont'd.

• Sketches of 2D Haar wavelet basis function projections:

PARALLEL PROJECTION



• Observations (PARALLEL projection):

- Each Haar wavelet basis function $H_{i,j}(x,y)$ defined by TWO values (+ and −) such that $\| H_{i,j}(x,y) \| = 1$.
- These projections are PIECEWISE LINEAR functions, since they are defined by the integrals $\left(\int_a^b H_{i,j}(\ell)\, d\tilde{\ell} \right)(t)$.
- Linear segments of these projections are defined by computing line integrals passing through corner vertices of "cells of $H_{i,j}$" that define $H_{i,j}$'s non-zero part in xy-space.
- A BEST APPROXIMATION in the Haar wavelet basis, i.e., $\sum_j \sum_i c_{i,j} H_{i,j}(x,y)$ has the same number of coefficients as a BOX basis function approximation and it supports a MULTI-RESOLUTION ANALYSIS.

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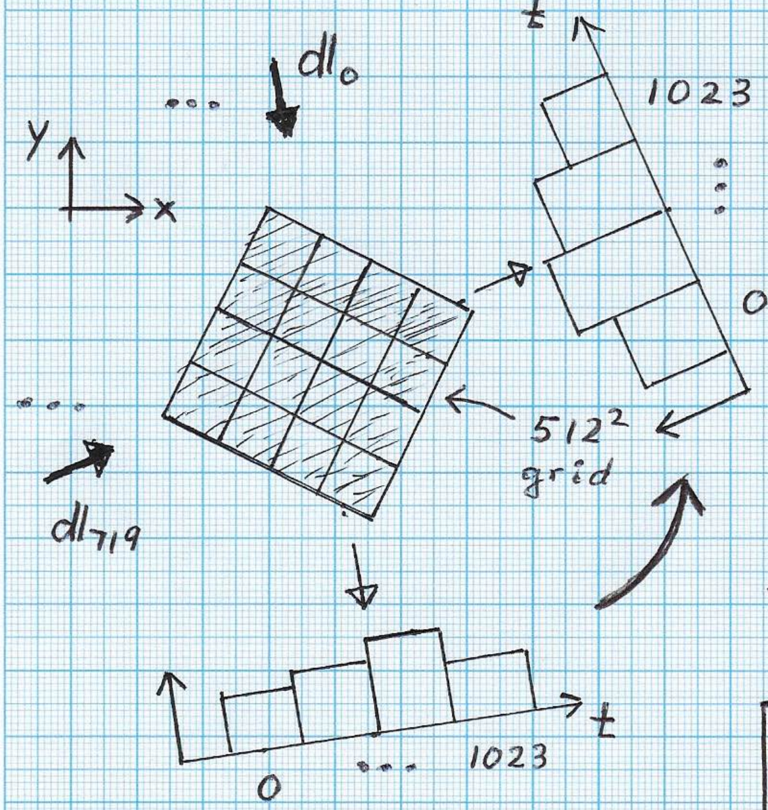
■ WAVELET RECONSTRUCTION – Cont'd.

→ "Discrete setting" (parallel projection):



• Example:

- 720 projection direction dl_i
- 1024 measured values per direction
- 512-by-512 reconstruction
- System to be solved to define $\sum_{j=0}^{511} \sum_{i=0}^{511} c_{i,j} H_{i,j}(x,y)$:

$$\begin{bmatrix} \langle \bar{I}_{0,0}, \bar{I}_{0,0} \rangle & \cdots & \langle \bar{I}_{0,0}, \bar{I}_{511,511} \rangle \\ \vdots & & \vdots \\ \langle \bar{I}_{511,511}, \bar{I}_{0,0} \rangle & \cdots & \langle \bar{I}_{511,511}, \bar{I}_{511,511} \rangle \end{bmatrix} \cdot \begin{bmatrix} c_{0,0} \\ \vdots \\ c_{511,511} \end{bmatrix} = \begin{bmatrix} \langle \bar{I}, \bar{I}_{0,0} \rangle \\ \vdots \\ \langle \bar{I}, \bar{I}_{511,511} \rangle \end{bmatrix},$$

abbreviated as

$$\boxed{\underset{512^2 \times 512^2}{H} \cdot \underset{512^2 \times 1}{\mathbb{C}} = \underset{512^2 \times 1}{I}}$$

• **BUT:** $\bar{I}$ "captures only 1024 pieces of information"

⇒ "Concatenate 720 systems:

$$720 \times 512^2 \left\{ \underbrace{\begin{bmatrix} H_0 \\ \vdots \\ H_{719} \end{bmatrix}}_{512^2} \right. \mathbb{C} = \left. \begin{bmatrix} I_0 \\ \vdots \\ I_{719} \end{bmatrix} \right\} 720 \times 512^2$$

⇒ least squares solution for $\mathbb{C}$!

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▪ WAVELET RECONSTRUCTION – Cont'd.

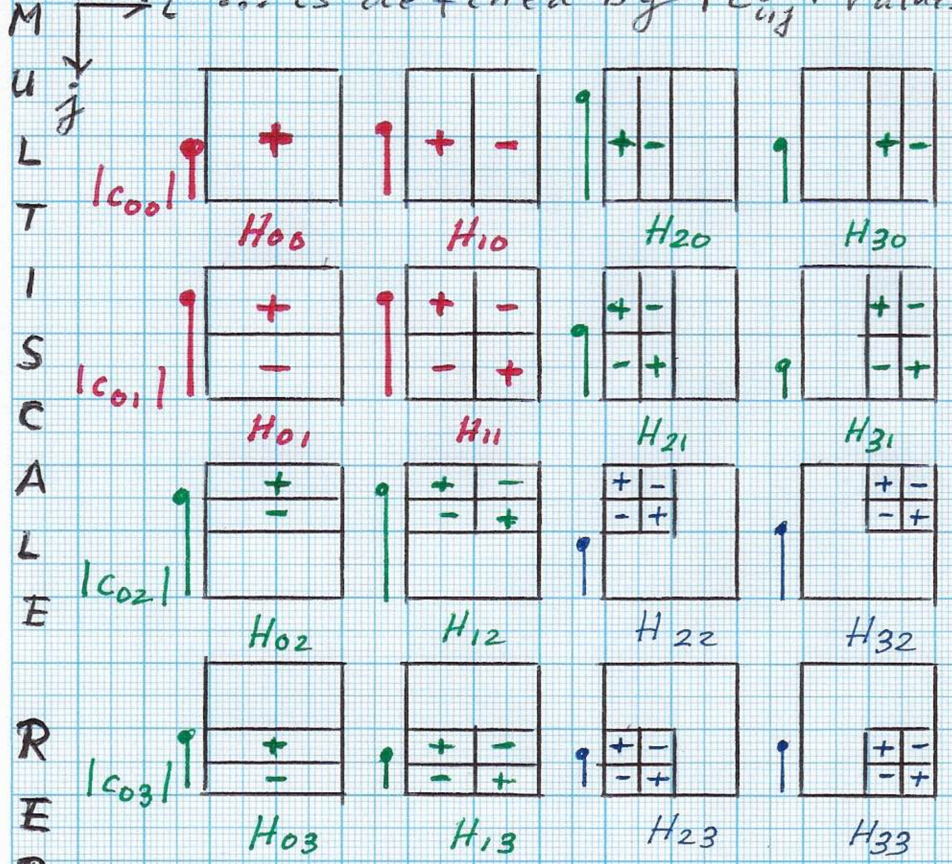
➜ Potential advantages of (Haar) wavelet reconstruction:

1) PROGRESSIVE RECONSTRUCTION

... makes possible the incremental, possibly adaptive, comutation of 64^2, 128^2, 256^2, 512^2 2D reconstructions – simply by considering more and more small-scale Haar wavelet basis functions $H_{i,j}(x,y)$.

2) SCALE-SPECIFIC BEHAVIOR

... is defined by $|c_{i,j}|$ values in $\sum_j \sum_i c_{i,j} H_{i,j}(x,y)$.

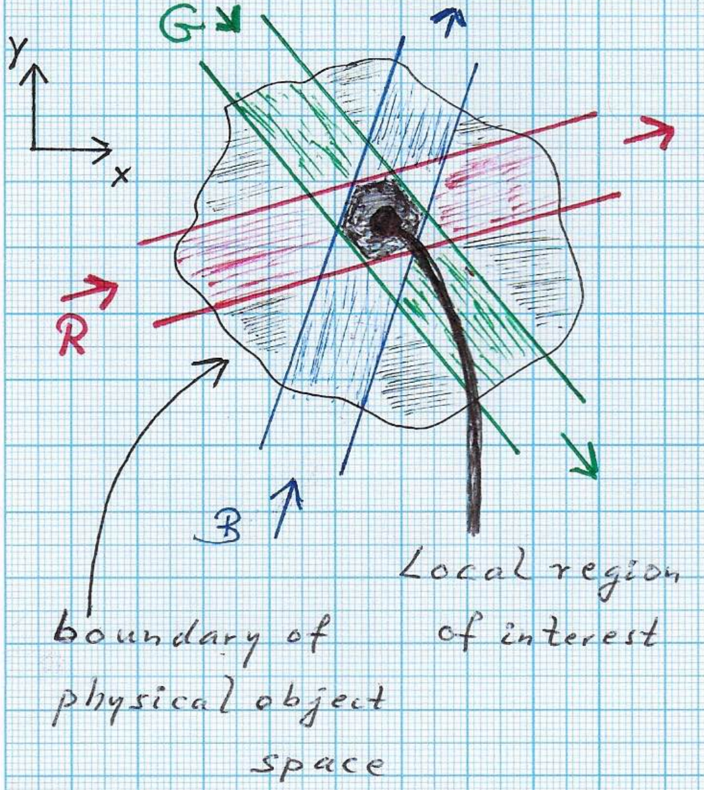


Red: Scale 1, Green: Scale $\frac{1}{2}$, Blue: Scale $\frac{1}{4}$

The values $|c_{i,j}|$ can be used as feature vector components, characterizing behavior at scales $1, \frac{1}{2}, \frac{1}{4}, \ldots$ Hypothesis: Histograms of values $|c_{i,j}|$ for specific scales permit classification of objects (density and/or geometry).

WAVELET RECONSTRUCTION – Cont'd.

3) LOCAL RECONSTRUCTION



... can be done in a "light cone intersection area. To perform reconstruction in the local region of interest (at some specific resolution) one must use all Haar wavelet basis functions that partially intersect with the local region of interest.

4) DATA COMPRESSION & NOISE REDUCTION

... are possible via a Haar wavelet representation by

(i) not considering functions $H_{i,j}(x,y)$ with coefficients $|c_{i,j}| < \varepsilon$ (⇒ compression)

and

(ii) not considering functions $H_{i,j}(x,y)$ that – together with noise-specific coefficients $c_{i,j}$ – represent noise (⇒ noise reduction)

~BH