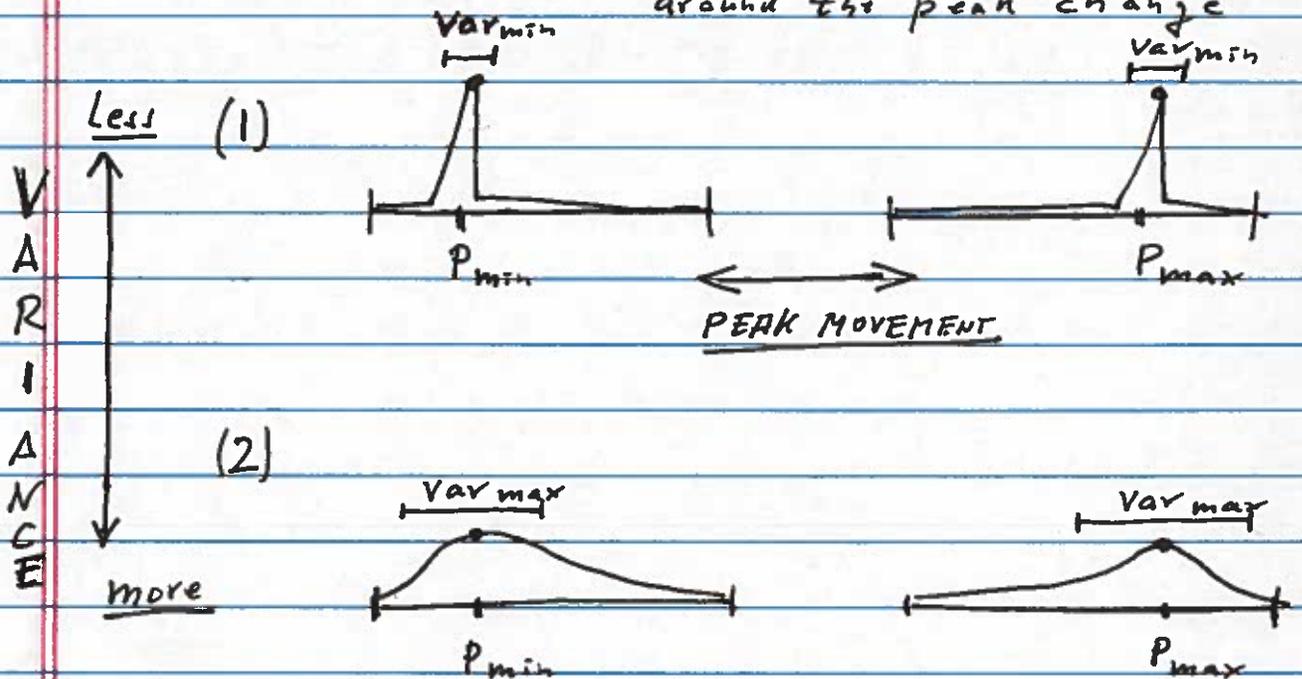


Bernd Hamann

- "Capturing the Variability of Histograms -  
The Same Object/Material Can Produce  
an Entire 'class' of Histograms"

- ➔ Simple Example: An object/material produces histograms where (1) the PEAK LOCATION and (2) the VARIANCE around the peak change



- ➔ How to capture and "model" this variation?

Here, one has two variables,

$x_1 = \text{peak location,}$

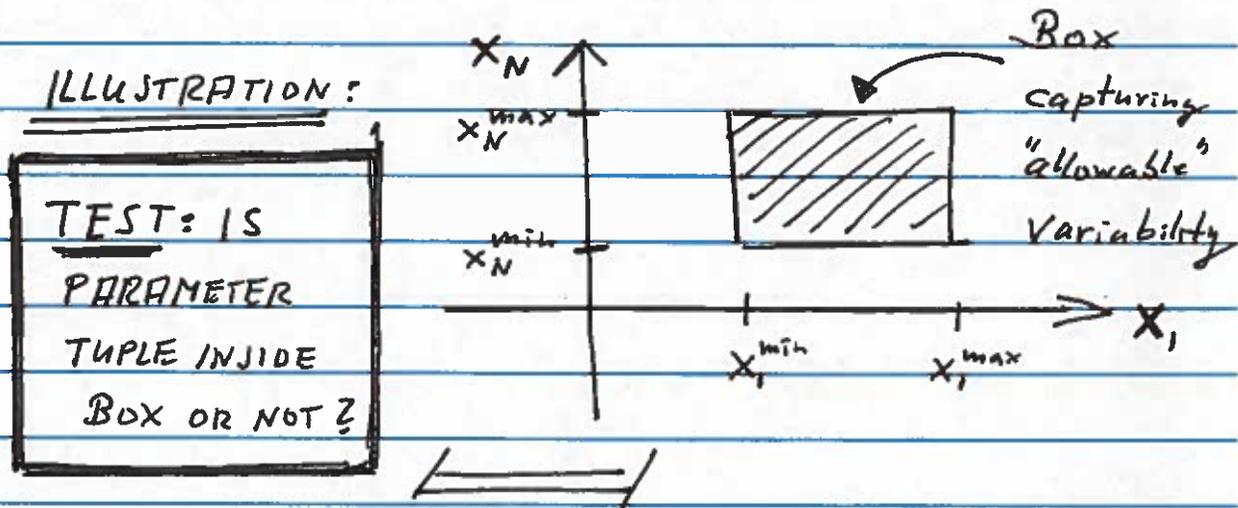
$x_1 \in [P_{min}, P_{max}]$ , and

$x_2 = \text{variance (around peak),}$

$x_2 \in [var_{min}, var_{max}]$

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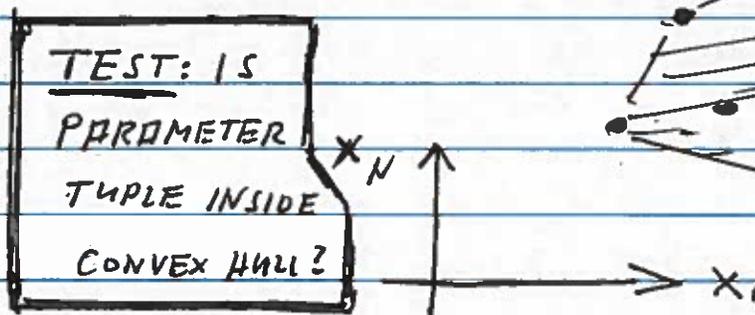
→ THUS: IF THERE ARE  $N$  VARIABLES FOR WHICH ONE "KNOWS" THEIR ALLOWABLE RANGES, THEN A "CLASS OF HISTOGRAMS" IS DEFINABLE VIA THE BOUNDING BOX GIVEN BY THESE RANGES



- BUT: A rectilinear bounding box might be "too large" and could include values of the  $N$ -dimensional tuple  $(x_1, \dots, x_N)$  that DO NOT represent a possible/allowable histogram of a specific object/material.
  - THUS: Should represent the allowable region via a "tighter" geometric definition

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→ Use CONVEX HULL of a known/given/measured/observed set of actually occurring values of a parameter tuple  $(x_1, \dots, x_N)$  to define an allowable region...

ILLUSTRATION:

- specific tuple for a histogram of an object/material

≡

- BUT: Two additional issues/problems arise:
- (i) Need to "enlarge" a bounding region by an " $\epsilon$ -strip" to handle noise etc.:



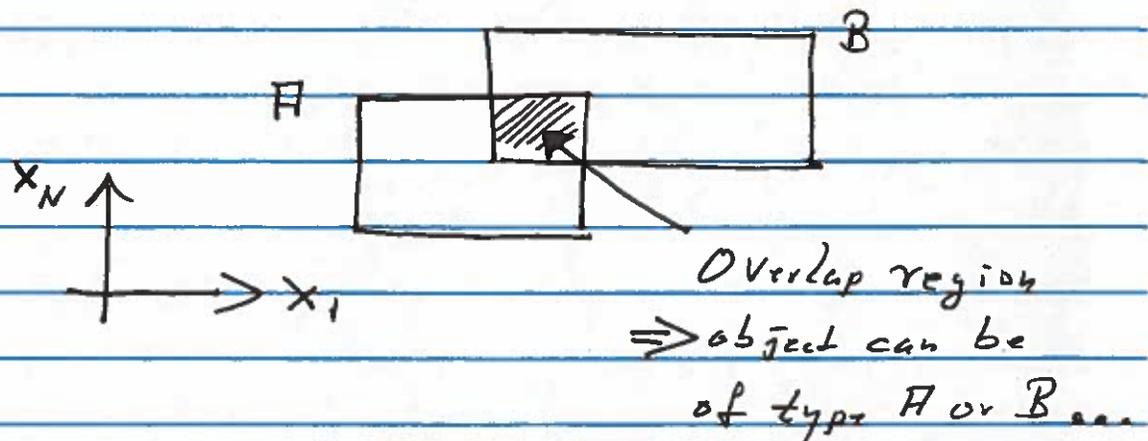
- (ii) Need to handle objects/materials with OVERLAPPING BOUNDING REGIONS...

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→ BIG PROBLEM: (ii), i.e., being able to distinguish objects/materials whose histograms could belong to multiple classes of histograms.

OR: Two different objects/materials might produce histograms that belong to two histogram classes that "overlap" - and we cannot decide to what class these two different objects/materials belong to

ILLUSTRATION: Two histogram classes that have associated parameter tuple regions that overlap:



→ HOW TO RESOLVE AMBIGUITY?

... → How to resolve the ambiguity when histograms can indicate a type-A or type-B material?

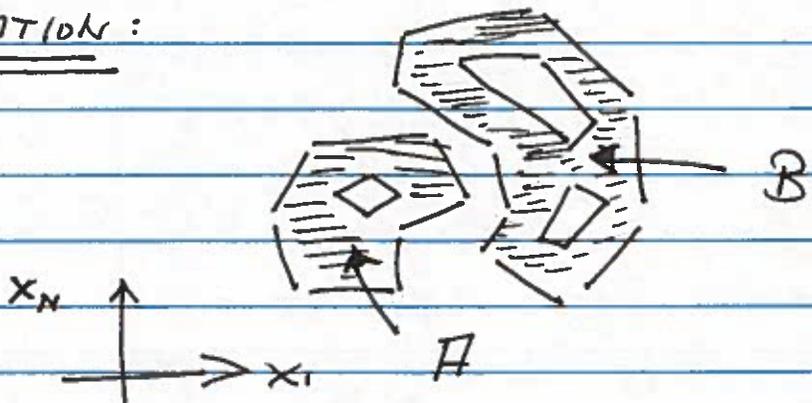
(i) Use more and more restrictions / conditions / available image parameters to decide to what type an object belongs

(ii) Use histogram-class regions that are even more restrictive than convex hulls:

→ allow non-convex regions

→ allow regions with holes (when allowable)

ILLUSTRATION:



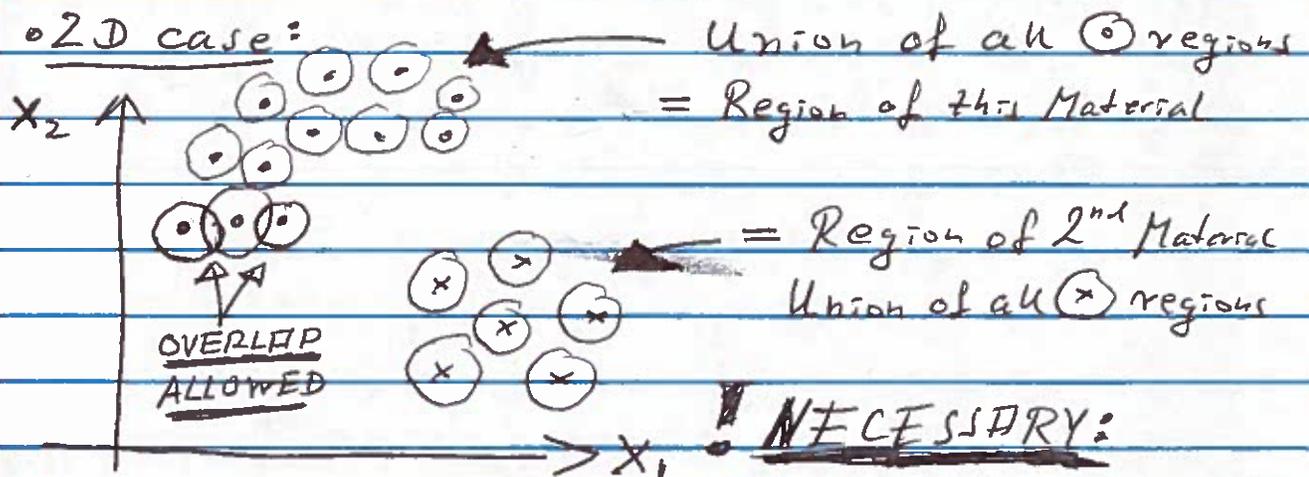
BUT: How to effectively and efficiently model / describe the regions defining type A and B?  
==

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- Another way to define the regions in space implied by given/known histogram characteristics of a material type/class:

→ UNION OF BALLS (related to  $\alpha$ -shapes)

- Concept: Given an observed set of parameter tuples  $(x_1, \dots, x_n)$  of the same material (imaged under different conditions), define the union of all balls of a certain radius and tuples  $(x_1, \dots, x_n)$  as the "Region of the Material".



$\odot$  and  $\otimes$  must not intersect to be able to differentiate two materials

- Issues: (1) How many tuples/balls needed per material?  
(2) How to define a radius (varying) for balls?