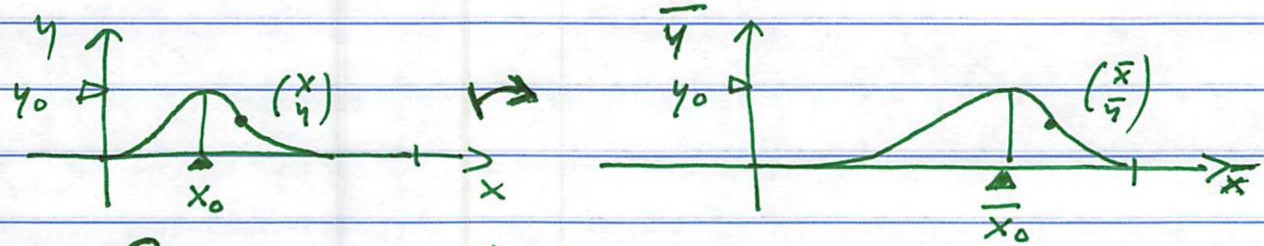
- Bernd Hamann, Stratovan 6/29/2015

- **Problem:** HISTOGRAM Transformation(s)



- **Basic transformations:**

① transl: $T: \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} + \begin{pmatrix}t_x\\ t_y\end{pmatrix}$

② scale: $S: \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}s_x & 0\\ 0 & s_y\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

③* reflX: $R_x: \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

reflY: $R_y: \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}$

[* Can be modeled via ②!]

- Parameters to determine: t_x, t_y, s_x, s_y; "yes/no": R_x, R_y

- Ex. (typical): 1^{st}: translate by $-x_0$
  2^{nd}: scale by s_x
  3^{rd}: translate by $+\bar{x}_0$

$\Rightarrow 1^{st}: \begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}x - x_0\\ y\end{pmatrix}$

$2^{nd}: \begin{pmatrix}\bar{\bar{x}}\\ \bar{\bar{y}}\end{pmatrix} = \begin{pmatrix}s_x & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}x - x_0\\ y\end{pmatrix} = \begin{pmatrix}s_x(x - x_0)\\ y\end{pmatrix}$

$3^{rd}: \begin{pmatrix}\hat{x}\\ \hat{y}\end{pmatrix} = \begin{pmatrix}s_x(x - x_0) + \bar{x}_0\\ y\end{pmatrix}$

here: x_0, $\bar{x}_0$ known (?)

s_x to be determined:

// $s_x = \dfrac{\hat{x} - \bar{x}_0}{x - x_0}$ // $\Longleftrightarrow$ OR VIA LEAST SQUARES!

- General **LEAST SQUARES** approach:

1^{st}: $\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}$

2^{nd}: $\begin{pmatrix} \bar{\bar{x}} \\ \bar{\bar{y}} \end{pmatrix} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix} = \begin{pmatrix} s_x (x - x_0) \\ s_y (y - y_0) \end{pmatrix}$

3^{rd}: $\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} s_x (x - x_0) \\ s_y (y - y_0) \end{pmatrix} + \begin{pmatrix} \bar{x}_0 \\ \bar{y}_0 \end{pmatrix}$

$$= \underline{\underline{\begin{pmatrix} s_x (x - x_0) + \bar{x}_0 \\ s_y (y - y_0) + \bar{y}_0 \end{pmatrix}}}$$

⇒ Define overdetermined system of equations:

$$\begin{pmatrix} \hat{x}_i \\ \hat{y}_i \end{pmatrix} = \begin{pmatrix} s_x (x_i - x_0) + \bar{x}_0 \\ s_y (y_i - y_0) + \bar{y}_0 \end{pmatrix},$$

$$i = 1, \ldots, n$$

⇒ must identify/define $\underline{(n+1)}$

corresponding point pairs:

$$\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \mapsto \begin{pmatrix} \bar{x}_0 \\ \bar{y}_0 \end{pmatrix},$$

$$\begin{pmatrix} x_i \\ y_i \end{pmatrix} \mapsto \begin{pmatrix} \hat{x}_i \\ \hat{y}_i \end{pmatrix}, \quad i = 1, \ldots, n$$

~ BH

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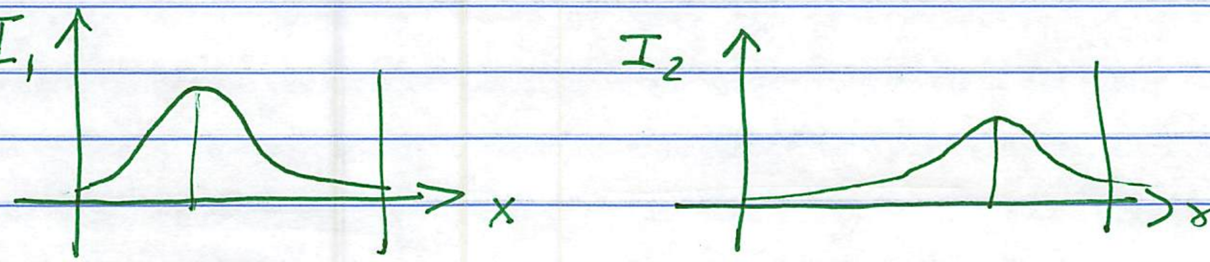
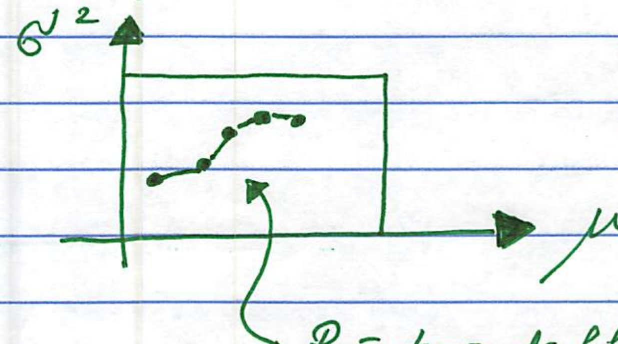
7/6/2015

- Using the empirically defined (normal) distribution implied by histograms of scans of the same material type to recognize a given material?



$\Rightarrow$ Each distribution of intensity values x defined by <u>mean</u>, <u>variance</u> (μ, σ^2)

$\Rightarrow$ Can represent each distribution by a point (μ, σ^2) of its defining 2 parameters:



"μ-σ^2-plane"

Points = different "allowable" histograms/distributions of the <u>same material</u> (imaged under <u>different conditions</u>)

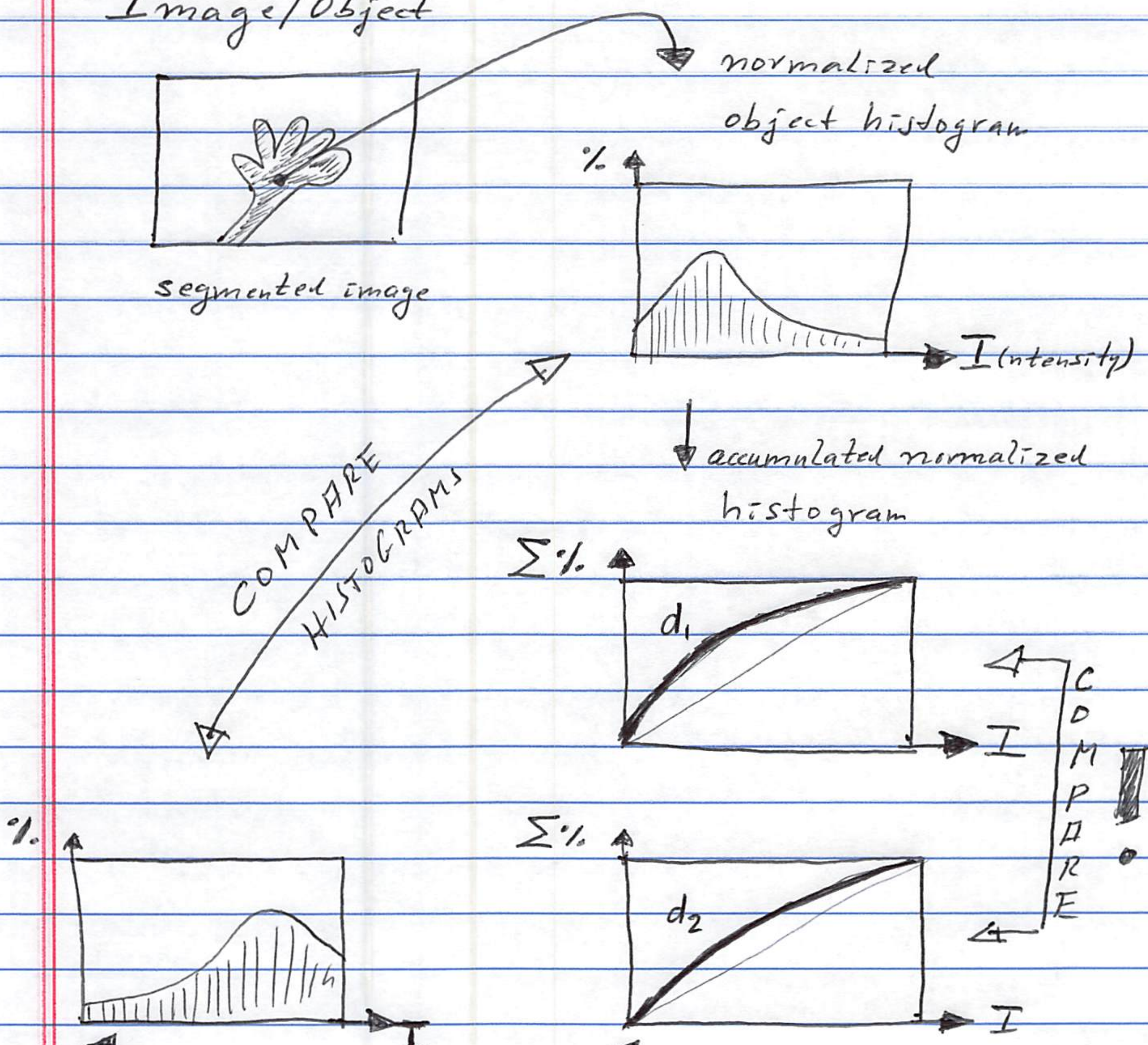
- <u>BUT</u>:

<u>One should probably NOT assume</u>/prescribe a "normal" <u>distribution model</u> - but a more "general" model...

WHICH ONE?

Use of HISTOGRAM EQUALIZATION and HISTOGRAM MATCHING for Detection:

Image/Object



normalized histogram for "MATERIAL A" type (from knowledgebase)

accumulated normalized histogram for "MATERIAL A" type (**NEEDED:** "distance(d_1, d_2)")

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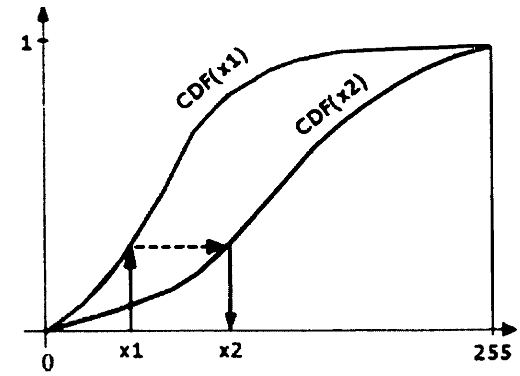
Histogram matching

From Wikipedia, the free encyclopedia

Histogram matching is a method in image processing of color adjustment of two images using the image histograms.

It is possible to use histogram matching to balance detector responses as a relative detector calibration technique. It can be used to normalize two images, when the images were acquired at the same local illumination (such as shadows) over the same location, but by different sensors, atmospheric conditions or global illumination.

• cumulative distributions



An example of histogram matching

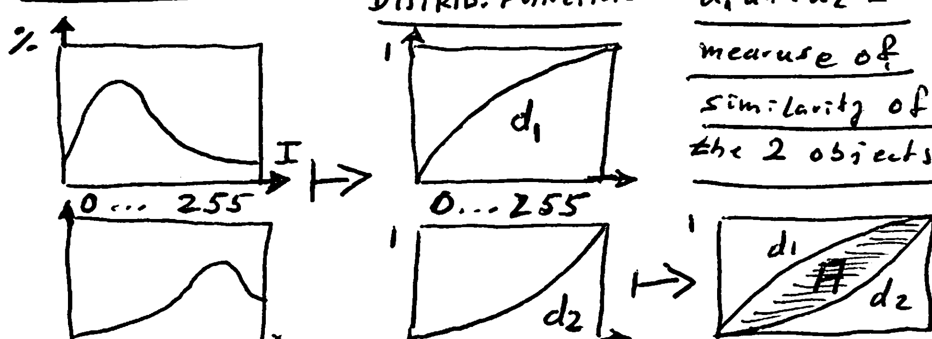
• 2 OBJECT SCANS

⇓

• 2 HISTOGRAMS

• 2 CUMULATIVE DISTRIB. FUNCTIONS

• "Area between d_1 and d_2 = measure of similarity of the 2 objects"



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The algorithm

Given two images, the reference and the adjusted images, we compute their histograms. Following, we calculate the cumulative distribution functions of the two images' histograms - $F_1()$ for the reference image and $F_2()$ for the target image. Then for each gray level $G_1 \in [0, 255]$, we find the gray level G_2 for which $F_1(G_1) = F_2(G_2)$, and this is the result of histogram matching function: $M(G_1) = G_2$. Finally, we apply the function $M()$ on each pixel of the reference image.

⇓ $A < \varepsilon$ ⇒ 2 objects of 'same' material

Multiple Histograms Matching

The Histogram matching Algorithm can be extended to find a monotonic mapping between two sets of histograms. Given two sets of histograms $P = \{p_i\}_{i=1}^k$ and $Q = \{q_i\}_{i=1}^k$, the optimal monotonic color mapping M is calculated to minimize the distance between the two sets simultaneously, namely $min_M \sum_k d(M(p_k), q_k)$ where $d(.,.)$ is a distance metric between two histograms. The optimal solution is calculated using dynamic programming [1]

References

Histogram equalization (normalization)

From Wikipedia, the free encyclopedia

Histogram equalization is a method in image processing of contrast adjustment using the image's histogram.

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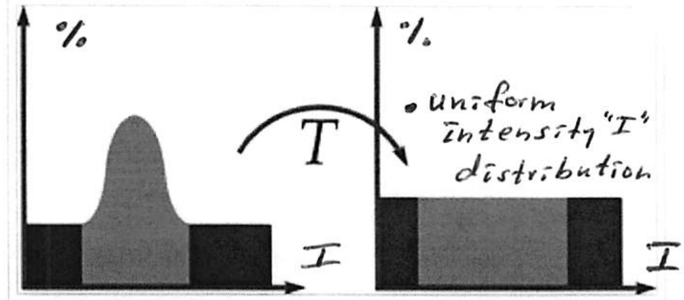


Histograms of an image before and after equalization.

• Question: Could it be beneficial to apply equalization to object's histogram(s) first – prior to histogram comparison / matching?

Overview

This method usually increases the global contrast of many images, especially when the usable data of the image is represented by close contrast values. Through this adjustment, the intensities can be better distributed on the histogram. This allows for areas of lower local contrast to gain a higher contrast. Histogram equalization accomplishes this by effectively spreading out the most frequent intensity values.

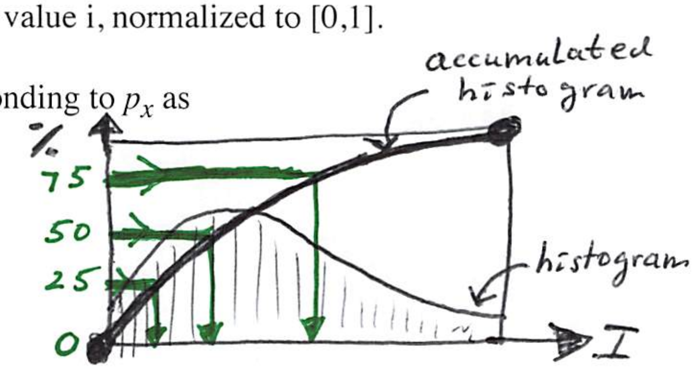
The method is useful in images with backgrounds and foregrounds that are both bright or both dark. In particular, the method can lead to better views of bone structure in x-ray images, and to better detail in photographs that are over or under-exposed. A key advantage of the method is that it is a fairly straightforward technique and an invertible operator. So in theory, if the histogram equalization function is known, then the original histogram can be recovered. The calculation is not computationally intensive. A disadvantage of the method is that it is indiscriminate. It may increase the contrast of background noise, while decreasing the usable signal.

In scientific imaging where spatial correlation is more important than intensity of signal (such as separating DNA fragments of quantized length), the small signal to noise ratio usually hampers visual detection.

Histogram equalization often produces unrealistic effects in photographs; however it is very useful for scientific images like thermal, satellite or x-ray images, often the same class of images to which one would apply false-color. Also histogram equalization can produce undesirable effects (like visible image gradient) when applied to images with low color depth. For example, if applied to 8-bit image displayed with 8-bit gray-scale palette it

L being the total number of gray levels in the image (typically 256), n being the total number of pixels in the image, and $p_x(i)$ being in fact the image's histogram for pixel value i, normalized to [0,1].

Let us also define the *cumulative distribution function* corresponding to p_x as



$$cdf_x(i) = \sum_{j=0}^{i} p_x(j),$$

which is also the image's accumulated normalized histogram.

We would like to create a transformation of the form $y = T(x)$ to produce a new image $\{y\}$, with a flat histogram. Such an image would have a linearized cumulative distribution function (CDF) across the value range, i.e.

$$cdf_y(i) = iK$$

for some constant K. The properties of the CDF allow us to perform such a transform (see Inverse distribution function); it is defined as

$$cdf_y(y') = cdf_y(T(k)) = cdf_x(k)$$

where k is in the range [0,L). Notice that T maps the levels into the range [0,1], since we used a normalized histogram of {x}. In order to map the values back into their original range, the following simple transformation needs to be applied on the result:

$$y' = y \cdot (\max\{x\} - \min\{x\}) + \min\{x\}$$

A more detailed derivation is provided here (http://www.math.uci.edu/icamp/courses/math77c/demos/hist_eq.pdf).

Histogram equalization of color images

The above describes histogram equalization on a grayscale image. However it can also be used on color images by applying the same method separately to the Red, Green and Blue components of the RGB color values of the image. However, applying the same method on the Red, Green, and Blue components of an RGB image may yield dramatic changes in the image's color balance since the relative distributions of the color channels change as a result of applying the algorithm. However, if the image is first converted to another color space, Lab color space, or HSL/HSV color space in particular, then the algorithm can be applied to the luminance or value channel without resulting in changes to the hue and saturation of the image.[4] There are several histogram equalization methods in 3D space. Trahanias and Venetsanopoulos applied histogram equalization in 3D color space[5] However, it results in "whitening" where the probability of bright pixels are higher than that of dark ones.[6] Han et al. proposed to use a new cdf defined by the iso-luminance plane, which results in uniform gray distribution.[7]

will further reduce color depth (number of unique shades of gray) of the image. Histogram equalization will work the best when applied to images with much higher color depth than palette size, like continuous data or 16-bit gray-scale images.

There are two ways to think about and implement histogram equalization, either as image change or as palette change. The operation can be expressed as *P(M(I))* where *I* is the original image, *M* is histogram equalization mapping operation and *P* is a palette. If we define a new palette as *P'=P(M)* and leave image *I* unchanged then histogram equalization is implemented as palette change. On the other hand if palette P remains unchanged and image is modified to *I'=M(I)* then the implementation is by image change. In most cases palette change is better as it preserves the original data.

Modifications of this method use multiple histograms, called subhistograms, to emphasize local contrast, rather than overall contrast. Examples of such methods include adaptive histogram equalization, *contrast limiting adaptive histogram equalization* or CLAHE, multipeak histogram equalization (MPHE), and multipurpose beta optimized bihistogram equalization (MBOBHE). The goal of these methods, especially MBOBHE, is to improve the contrast without producing brightness mean-shift and detail loss artifacts by modifying the HE algorithm.[1]

A signal transform equivalent to histogram equalization also seems to happen in biological neural networks so as to maximize the output firing rate of the neuron as a function of the input statistics. This has been proved in particular in the fly retina.[2]

Histogram equalization is a specific case of the more general class of histogram remapping methods. These methods seek to adjust the image to make it easier to analyze or improve visual quality (e.g., retinex)

Back projection

The **back projection** (or "project") of a histogrammed image is the re-application of the modified histogram to the original image, functioning as a look-up table for pixel brightness values.

For each group of pixels taken from the same position from all input single-channel images, the function puts the histogram bin value to the destination image, where the coordinates of the bin are determined by the values of pixels in this input group. In terms of statistics, the value of each output image pixel characterizes the probability that the corresponding input pixel group belongs to the object whose histogram is used.[3]

Implementation

Consider a discrete grayscale image $\{x\}$ and let n_i be the number of occurrences of gray level i. The probability of an occurrence of a pixel of level i in the image is

$$p_x(i) = p(x = i) = \frac{n_i}{n}, \quad 0 \le i < L$$